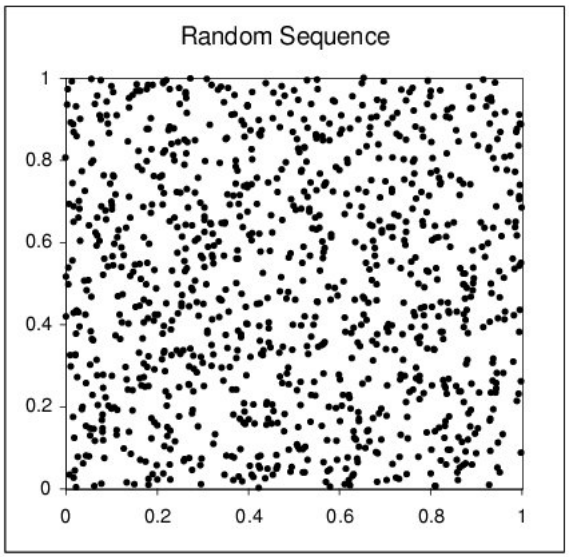
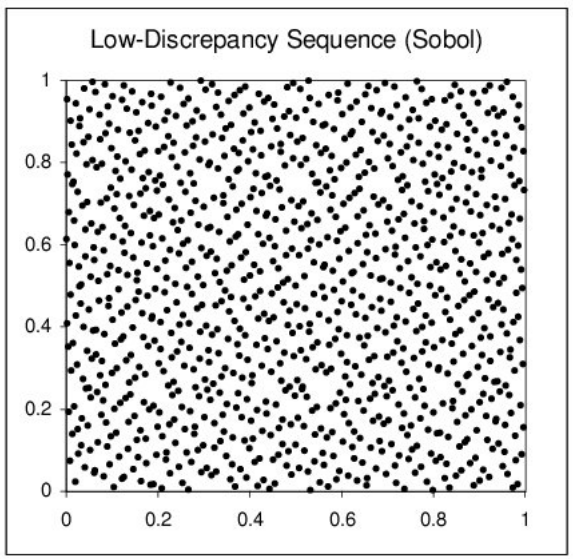
Quasi-Monte Carlo Integration for MTM3

Igor Volobouev

Texas Tech University

Quasi-Monte Carlo





- Based on the concept of deterministic “low-discrepancy sequences”
- Discrepancy is a measure of uniformity of a set of points. Several types of discrepancies have been studied. The most widely used is the “star discrepancy” which is a multivariate generalization of the Kolmogorov-Smirnov distance w.r.t. the uniform distribution:

$$D^*(P) = \sup_{B \in J^*} \left| \frac{1}{N} \sum_{i=1}^{N} c_B(\vec{x}_i) - S(B) \right|$$

P is the point set

*J** is the family of all subintervals of the d-dimensional unit cube of the form $\prod_{k=1}^{d} [0, u_k)$

c_B is the characteristic function of subinterval *B* (1 if the point is in *B*, 0 otherwise)

S(*B*) is the hypervolume of *B*

QMC Integration Convergence

- Koksma-Hlawka inequality: quasi-MC integration converges at least as fast as $O(\log(N)^d/N)$ for "well-behaved" functions. Compare with $O(1/\sqrt{N})$ convergence for standard MC.
- Note that this inequality provides a *deterministic upper bound* on the convergence rate. The *actual* convergence rate can be much better.
- QMC convergence for various types of integrands is still an active research area in applied mathematics

Example Quasi-MC Convergence Study

G. Takhtamyshev et al. / Mathematics and Computers in Simulation 73 (2007) 309–319

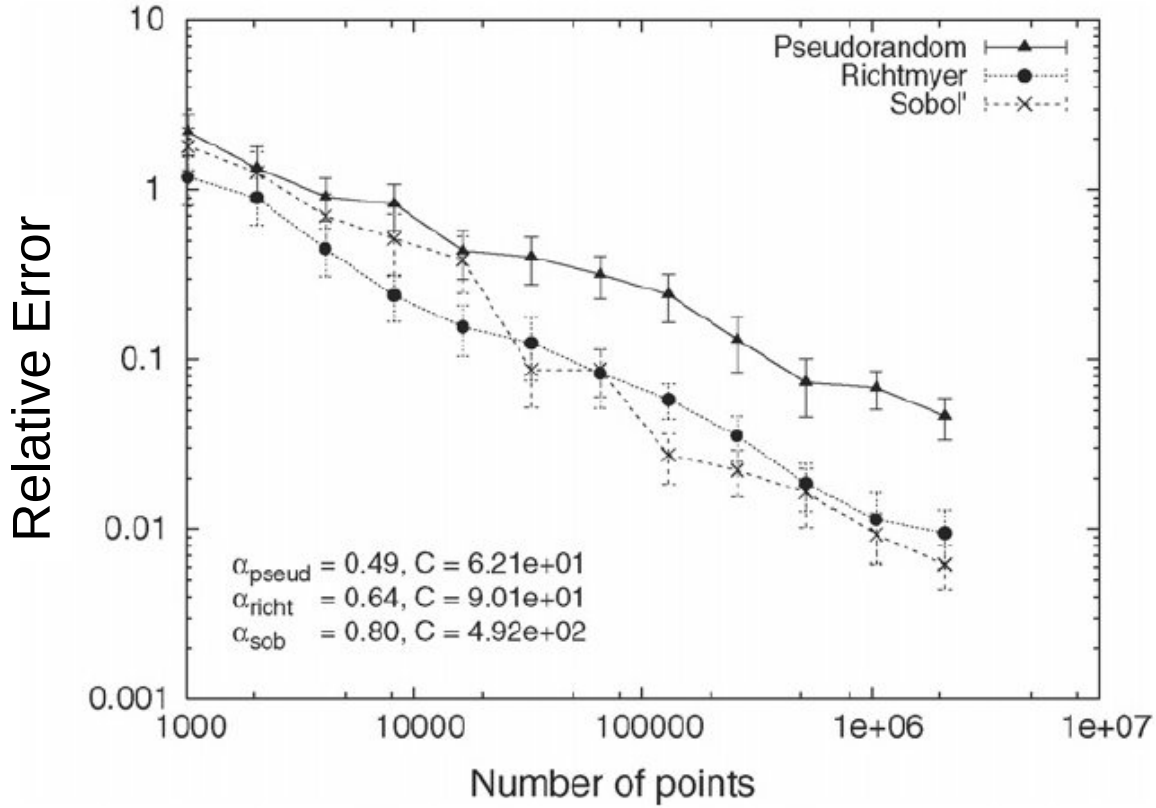
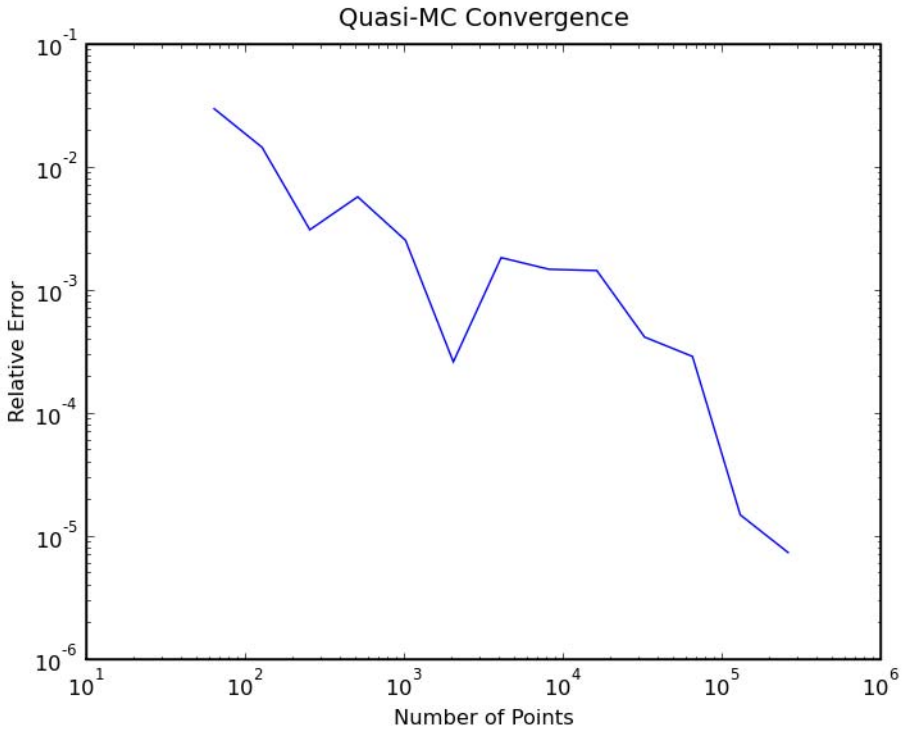


Fig. 1. Result for $f_1(\mathbf{x})$ in 100 dimensions ($a = 0.35$, $I \approx 10^{-5}$).

$$f_1(x_1, \ldots, x_s) = \prod_{i=1}^{s} \exp\left(-a_i |x_i - u_i|\right)$$

Simple HEP-Related Example: Muon Lifetime



- Calculated as a generic 3-body decay with known matrix element (5-d integral, Sobol sequence). Compared with the exact theoretical answer.

Extensive QMC convergence study for HEP-related integrals: R. Kleiss, A. Lazopoulos, *Computer Physics Communications* **175**, pp 93-115 (2006)

QMC Implementation in MTM3

- Based on TOMS algorithm 823: “Implementing Scrambled Digital Sequences” by H. Hong and F.Hickernell
- Integration variables are: M_{tl}, M_{th}, M_{Wl}, M_{Wh}, $\log(p_{q1}/p_{q2})$, p_{Tttbar}, and (η, φ, m) for each jet-producing parton
- Importance sampling is used for most of these variables
- 19-d integral overall, QMC is applied in 18 dimensions. Special integration procedure is used for the leptonic W mass/neutrino p_z variable to avoid the phase space singularity.
- Convergence is estimated empirically from the smoothness of likelihood curves