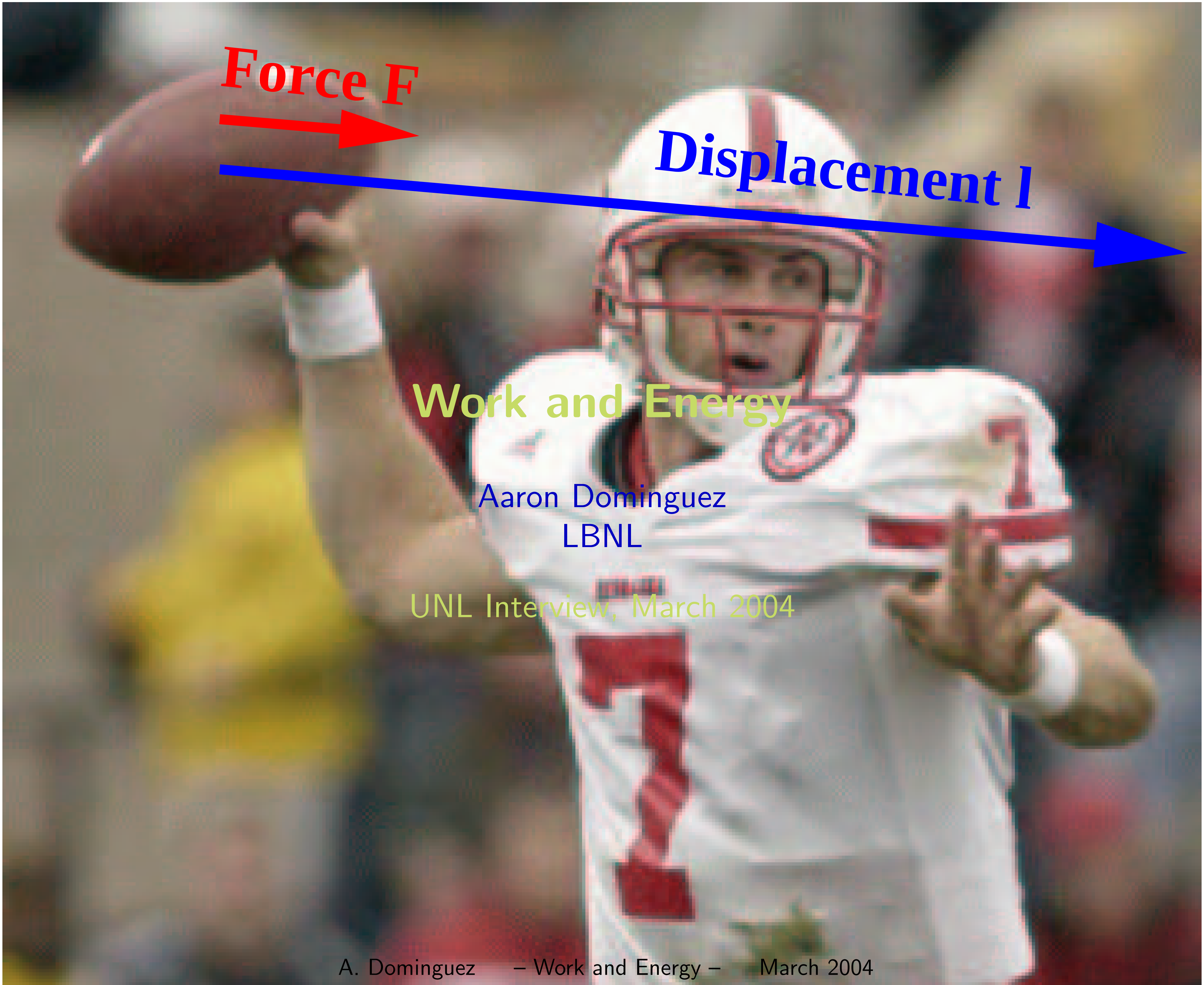

Work and Energy

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Energy & Work

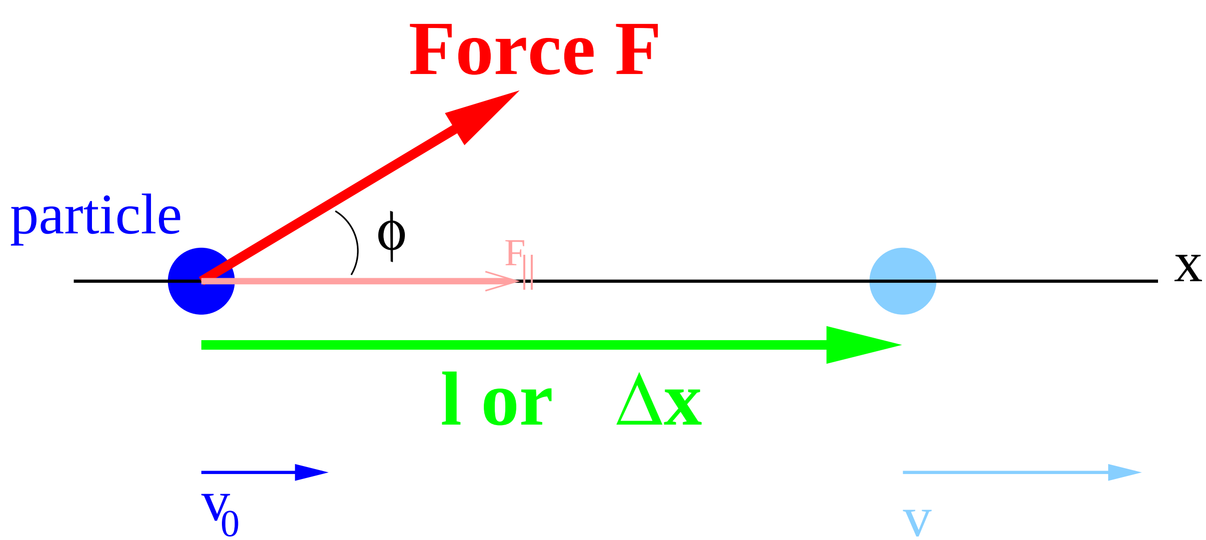
- Until now, studied translational motion using Newton's 3 laws
- **Force** played a central role in determining the motion of an object
- Let's consider alternate analysis of motion of objects in terms of (conserved quantity) **energy** and **work**

Applying Force Over a Distance

- In physics, work given specific meaning: *describes what's accomplished when applying a force to an object over a distance*

- In particular, work done by force constant in magnitude and direction is defined to be product of magnitude of force parallel to direction of displacement

- Units: $\mathrm{J} \equiv \mathrm{N} \cdot \mathrm{m}$, joules



$$\boxed{W = \mathbf{F} \cdot \mathbf{l}}$$ scalar, but signed

Can Be Tricky

- Easy to get confused unless we carefully consider if we're talking about *work done by* a specific object, or *on* a specific object

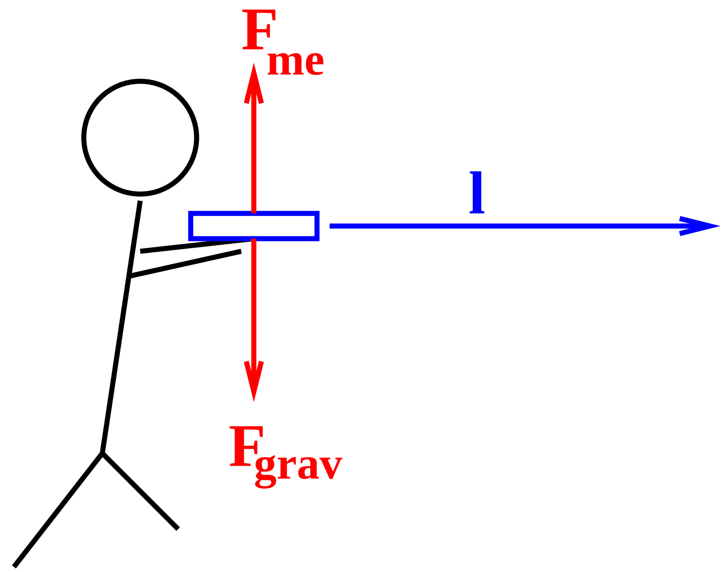
- Also important to consider if work is done due to *one particular* force, or the *net work* done by *net force* acting on object

- In general: **You have to move something by applying a force in the same/opposite direction as direction of motion**

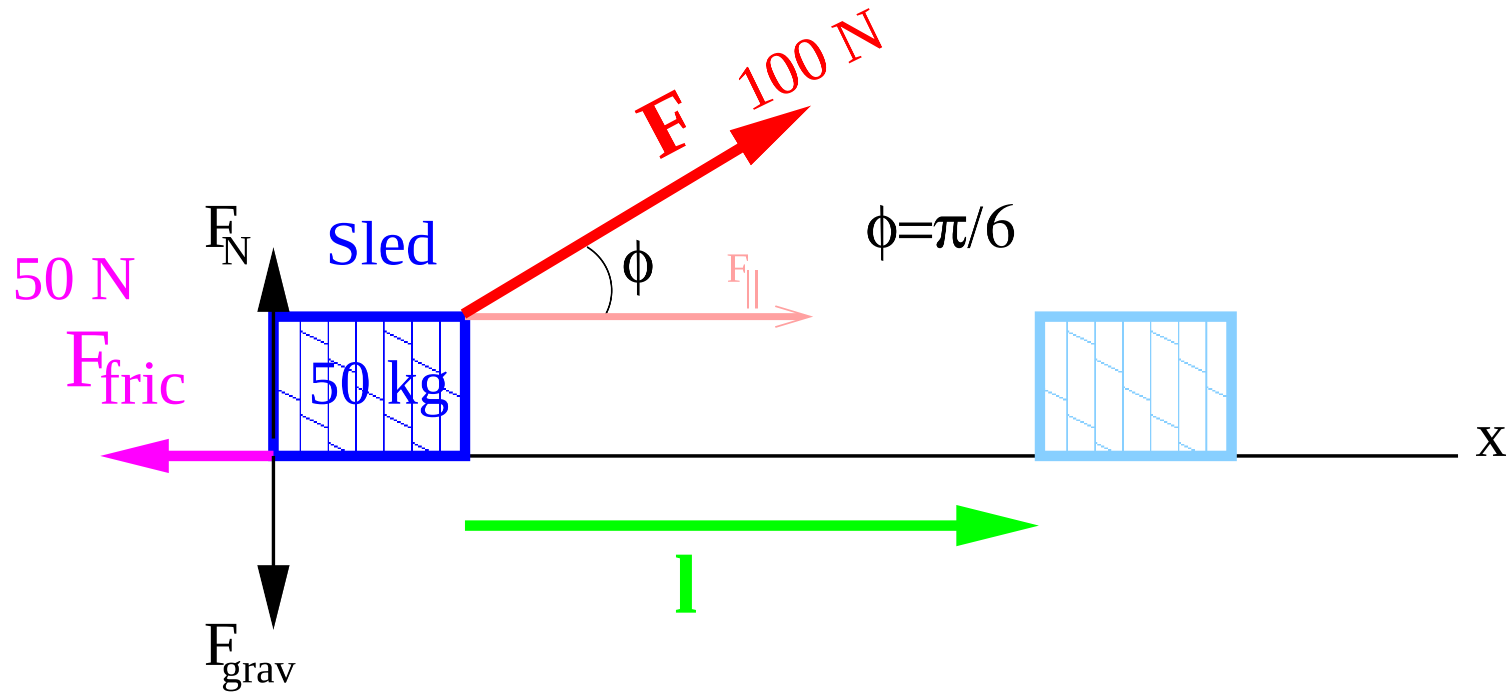
Holding Book Steady While Walking

- Holding book steady in my arms
- Walking forward without raising/lowering book
- Work done *on book by me* is:

1. Positive
2. Zero
3. Negative



Example: Work Done by Pulling a Sled



- Work done by each force:
  - * $W_{\text{grav}} = W_{\text{N}} = 0$
  - * $W_{\text{puller}} = \mathbf{F} \cdot \mathbf{l} = 500\sqrt{3}\text{J}$ (pos work)
  - * $W_{\text{fric}} = \mathbf{F}_{\text{fric}} \cdot \mathbf{l} = -500\text{J}$ (neg work)
- Net Work:
  - * $W_{\text{net}} = W_{\text{grav}} + W_{\text{N}} + W_{\text{puller}} + W_{\text{fric}} = 500(1 - \sqrt{3})\text{J}$

In general, work is *negative* if done by force acting in *direction opposite to motion*

General Definition of Work

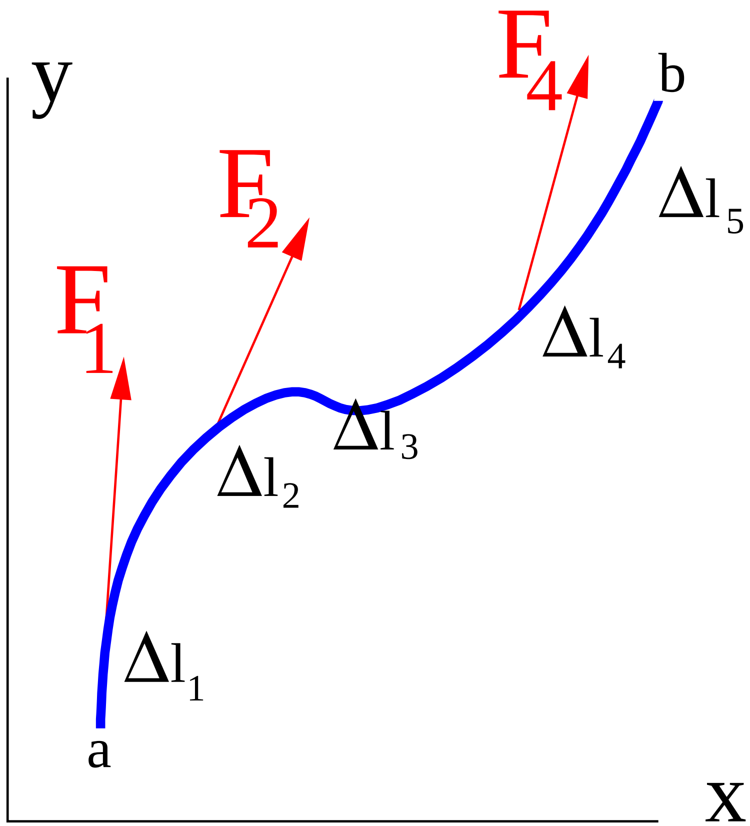
- For constant force and displacement, $W = \mathbf{F} \cdot \mathbf{l}$

- But what if $\mathbf{F}$ or $\mathbf{l}$ varies?

- Total work is sum of small intervals $\sum_i \mathbf{F}_i \cdot \mathbf{l}_i$

- In limit of small displacements,

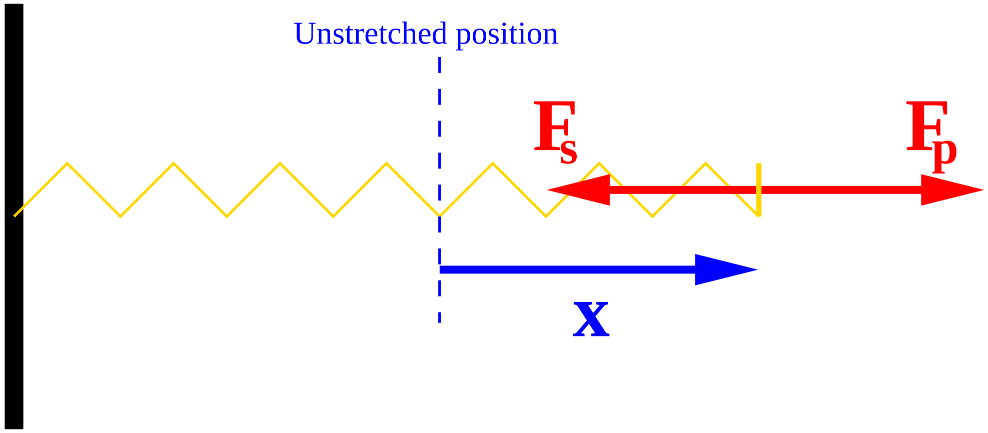
$$W = \int_a^b \mathbf{F} \cdot d\mathbf{l}$$



- If $\mathbf{F}$ is known analytically, then we can try integrating it analytically, otherwise use some numerical technique

- What are some forces that change as a function of displacement?

Example: Work Done by Stretching Ideal Spring



- Recall Hooke's law: $\mathbf{F}_s = -kx$ (k is spring constant)
- What is work done by person to slowly stretch spring?
- $\mathbf{F}_p = -\mathbf{F}_s = kx$
- Can solve analytically

$$W_p = \int_0^x \mathbf{F} \cdot d\mathbf{l} = \int_0^x kx\,dx = \frac{1}{2}kx^2|_0^x = \frac{1}{2}kx^2$$

- Could also solve graphically...

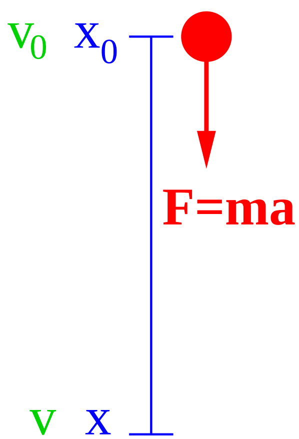
Kinetic Energy & Work-Energy Principle

- *Energy* is one of most important concepts in physics, but not easy to make a short but broad definition
- Consider *translational kinetic energy*
- For this mechanical energy, can define energy in canonical way as *"ability to do work"*
- Somewhat vague, but underscores fundamental connection between work and energy
- Object in motion has ability to do work

  $\Rightarrow$ Energy of motion is called kinetic energy

Translational Kinetic Energy

- Use our study of Newton's laws to understand connection between kinetic energy and work. How does gravity change velocity?



- Recall: $v^2 - v_0^2 = 2a\Delta x$

- $W_{\rm net} = F_{\rm net}(x - x_0) = ma(x - x_0)$

- Substitute $a = 1/2(v^2 - v_0^2)/(x - x_0)$

$$W_{\rm net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- $K \equiv \frac{1}{2}mv^2$ *defined to be translational kinetic energy*

$$\boxed{W_{\rm net} = K - K_0 = \Delta K}$$ Work-Energy Principle

- Net work done on an object is equal to its change in kinetic energy