Section 9

SYSTEMATIC UNCERTAINTIES

The general procedure for estimating the systematic uncertainty on M_{top} from a given source is handled similarly for all sources. For a given source of uncertainty, we make a change in the input value, and evaluate the impact on the measured top quark mass. The change is either a one standard deviation (1σ) uncertainty on the variable in question, or a change in an input assumption. The change in the top quark mass is evaluated using simulated experiments (see Section 7.3). We perform a large number of simulated experiments with (a) the nominal input value of the variable or the standard assumption, and (b) a "1σ" shift in the variable value or the changed assumption. The reconstructed-mass distribution from each simulated experiment is fit using the same likelihood procedure as used on the data sample, thus obtaining a measured top quark mass. The likelihood procedure includes the same templates as used with the data. The systematic uncertainty is defined as the difference in the median top quark mass between the two ensembles (a) and (b). The distribution of reconstructed top quark masses from simulated experiments in which all inputs are set to their nominal values was shown in Fig. 7.6.

9.1 Uncertainties from the energy scale

The event reconstruction algorithm varies the measured momenta of the jets, leptons, and unclustered energy to fit the kinematics of the hypothesized $t\bar{t}$ decay. The energy scale for electrons and the momentum scale for muons are known to a precision of 0.14% and 0.065% respectively [31]. This uncertainty has a negligible effect on the uncertainty in the top quark mass measurement. The uncertainty on the quantity X_T, the transverse energy beyond the partons associated with the $t\bar{t}$ event, has been discussed in Section 5.2. This uncertainty is large, but large variations of X_T do not have a significant impact on the mass reconstruction. The $\not{E}_{\mathrm{T}}$ is evaluated through the measurements of the charged lepton, the jets, and the unclustered energy, and is therefore not an independent measurement (see Section 5.2). To avoid correlations it is not used as a measurement in the fit, but is used to estimate a starting value for the transverse momentum of the neutrino. Thus, the energy scale uncertainty in the measured top quark mass is dominated by the uncertainty in the measurement of the jet momenta.

The total uncertainty in the jet $\mathrm{P_T}$ scale is taken as the quadrature sum of all uncertainties discussed in Section 5.1. We apply $+1\sigma$ and -1σ shifts to the jet momenta in $t\bar{t}$ signal and background events, and measure the effect on the measurement of the top quark mass. For the SVX Single subsample, the distributions of reconstructed masses for -1σ and $+1\sigma$ shifts in the $\mathrm{P_T}$ scale are shown in Figs. 9.1(a) and (c) respectively. These distributions may be compared to Fig. 9.1(b) which shows the distribution obtained from the default momentum scale. As expected, a clear shift in the reconstructed-mass spectrum is observed. We generate analogous distributions for the other three mass subsamples and for the background mass distribution. To obtain the systematic uncertainty, we generate two large samples of simulated experiments. In the first sample, we choose the reconstructed masses for $t\bar{t}$ events at random from distributions like the one in Fig. 9.1(a). In the second sample, we use distributions like the one in Fig. 9.1(c). The simulated experiments in each of these samples are fit using the standard templates and the likelihood technique described in Section 7. The median top mass from the simulated experiments in the two samples differ because of the applied jet $\mathrm{P_T}$ scale shifts. The distribution of reconstructed top masses from the two (jet $\mathrm{P_T}$ shifted) samples are displayed in Fig. 9.2. We take half the difference between the medians of the -1σ and $+1\sigma$ distributions (from Fig. 9.2) as the uncertainty on the top quark mass measurement due to the $\mathrm{P_T}$ scale uncertainty. Using this prescription, we obtain a top mass uncertainty of ± 4.4 GeV/c^2 from the jet $\mathrm{P_T}$ scale.

9.2 Initial and final state hard radiation

QCD radiation that produces jets can originate from the outgoing (final state) partons, the incoming (initial state) partons, or from interference among the two. The interference effect is expected to be small [56] and is not considered here.

The effects of initial state radiation (ISR) and final state radiation (FSR) on the measurement of M_{top} are studied using the PYTHIA program since it allows the two effects to be studied in isolation from one another. The approach used to evaluate the systematic uncertainty due to ISR is to compare the median mass from simulated experiments using the standard PYTHIA settings to the median mass from simulated experiments with ISR turned off. The median mass from simulated experiments for the no-ISR PYTHIA sample is found to be lower than that of the standard PYTHIA sample by 2.6 GeV/c^2. The uncertainty is taken to be one half of the shift in median mass between

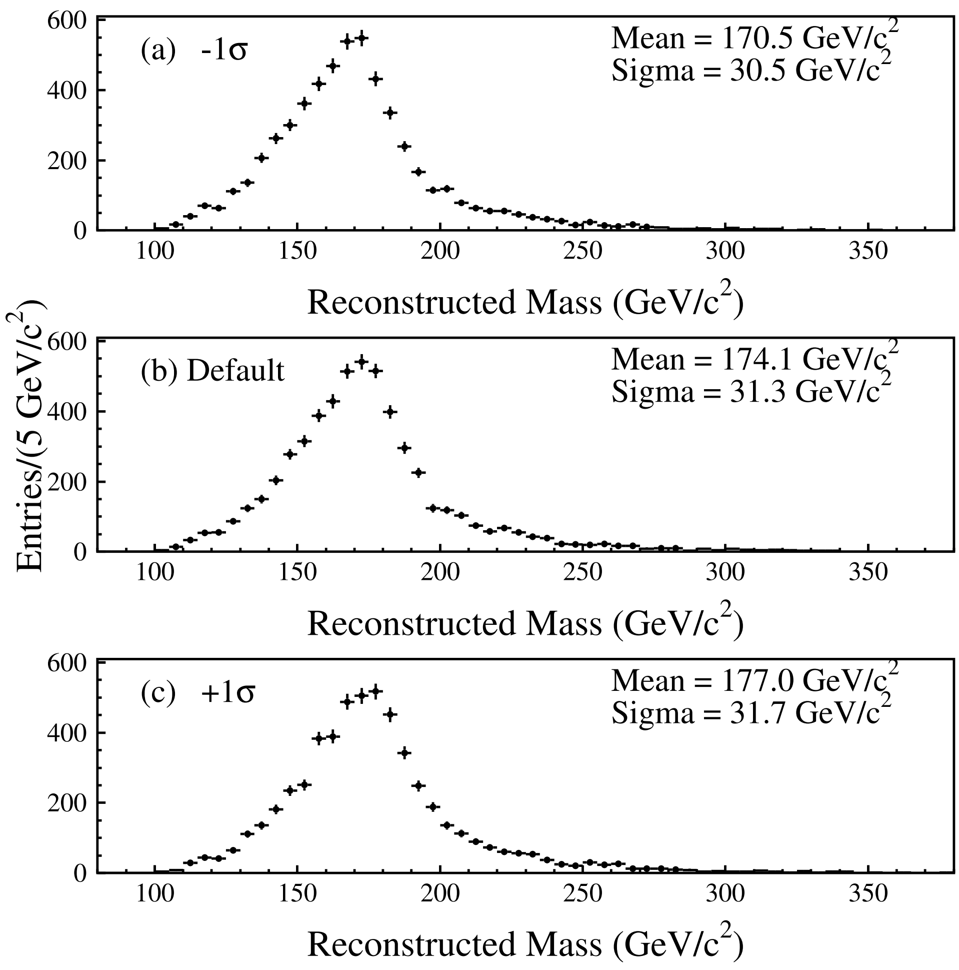

Figure 9.1: Distribution of reconstructed masses for SVX Single tagged events from the HERWIG $t\bar{t}$ Monte Carlo simulation using an input top mass 175 GeV/c^2 for (a) a -1σ shift in the jet $\mathrm{P_T}$ scale, (b) no shift in the jet $\mathrm{P_T}$ scale, and (c) a $+1\sigma$ shift in the jet $\mathrm{P_T}$ scale. These distributions are used as inputs to generate the samples of simulated experiments described in the text.

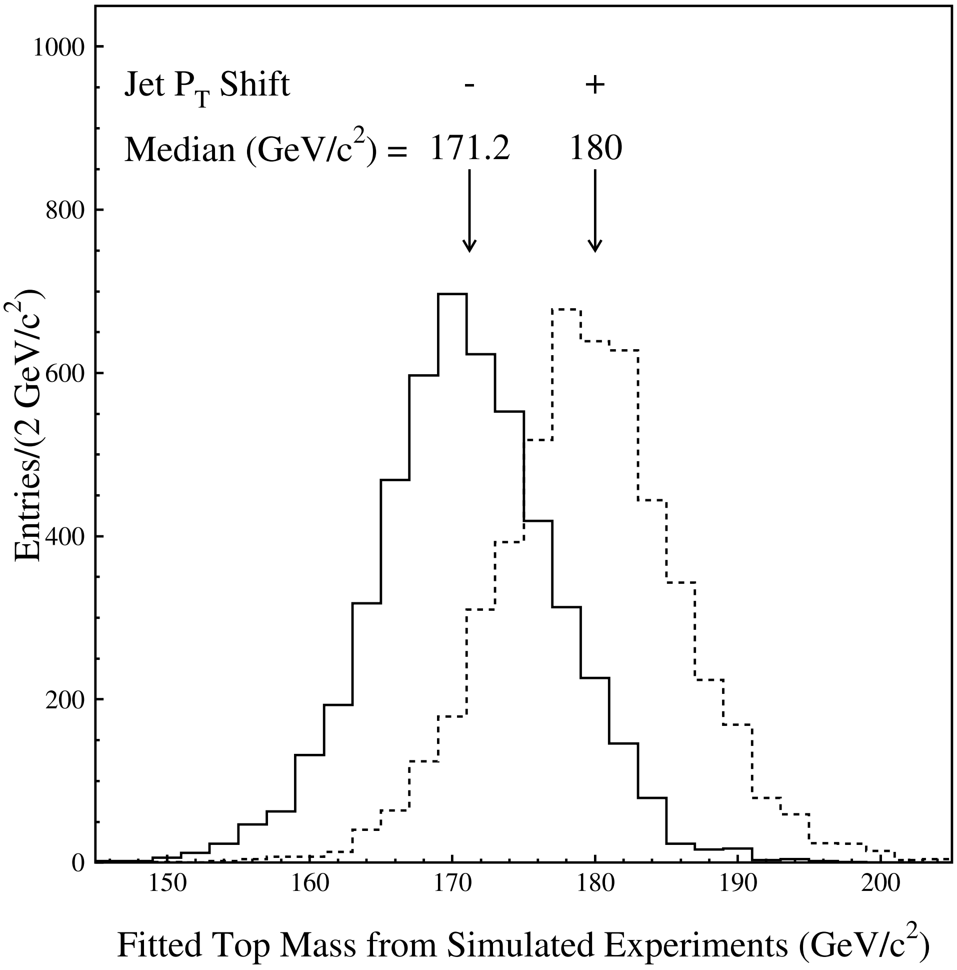

Figure 9.2: Distribution of fitted top masses returned from the likelihood procedure for simulated experiments. The solid histogram shows the distribution when the jet P_T scale is shifted down by -1σ, and the dashed histogram shows the results when the jet P_T scale is shifted up by $+1\sigma$. The median top masses for each are indicated on the figure, from which we obtain a systematic uncertainty of 4.4 GeV$/c^2$.

the standard PYTHIA simulation and the no-ISR PYTHIA simulation, which is 1.3 GeV/c^2. We assume that the shift is symmetric with the amount of ISR and therefore the uncertainty is ±1.3 GeV/c^2.

Extracting the effects due to final state radiation is a more subtle exercise because PYTHIA, like HERWIG, describes jet formation through a parton shower. The effects of modeling the softer components on the measurement of M_{top} are described by the studies of soft gluon radiation (see Section 5.1.1). In this discussion, we are therefore referring to the "harder" component of FSR, which leads to extra jets in the final state. To isolate the effect of FSR, we use a sample of PYTHIA events which have ISR turned off. We select a subsample of these events that have exactly four jets (either four high-E_T jets, $E_T > 15$ GeV and $|\eta| < 2.0$, or three high-E_T jets plus one with $E_T > 8$ GeV and $|\eta| < 2.4$), all of which can be uniquely matched to the partons from $t\bar{t}$ decay (within a cone of 0.4). Using the procedure of simulated experiments, we take the systematic uncertainty to be half the difference between the no-ISR PYTHIA events with no restriction on the number of jets and the subsample of events with exactly four jets uniquely matched to the partons from $t\bar{t}$ decay. We assume that this difference is approximately independent of the amount of ISR present. The median top mass from the no-ISR sample with exactly four jets is found to be larger than the default no-ISR sample by 4.4 GeV/c^2. The systematic uncertainty is taken to be one half of the difference in the median top masses between the two samples, or 2.2 GeV/c^2. As with ISR, we assume that the shift in top mass is symmetric with respect to the amount of FSR, so the systematic uncertainty is ±2.2 GeV/c^2.

9.3 Background mass distribution

In generating the default background distributions with the VECBOS program, we used the W+3 partons matrix elements and chose a scale of

$Q^2 = \langle \mathrm{P_T} \rangle^2$. This Q^2 scale is not only used in VECBOS for the computations of the matrix elements and the evaluation of the parton distribution functions, but it is also used in the evolution of the parton showers to limit the $\mathrm{P_T}$ of additional jets [59]. As a result, the shape of the reconstructed-mass distribution is sensitive to the choice of scale. The systematic uncertainty from using the assumed background shape was evaluated by changing the Q^2 scale from $\langle \mathrm{P_T} \rangle^2$ to M_W^2. Simulated experiments using $t\bar{t}$ and the modified background shape ($Q^2 = M_W^2$) were fit to the default signal and background probability distributions as described in Section 7. The median mass from these simulated experiments was found to differ by 1.3 GeV/c^2 from simulated experiments using the default background shape. The systematic uncertainty from the background shape modeling is taken to be this difference symmetrized, or ± 1.3 GeV/c^2.

9.4 b-Tagging

A systematic uncertainty in the top mass measurement may arise from an uncertainty in the SVX and SLT tagging efficiencies. For SVX tagging, the primary uncertainty comes from the possible $\mathrm{E_T}$ dependence of the SVX tagging efficiency which may differ from the simulation. The SVX tagging efficiency in data relative to the simulation is parametrized as a function of the jet $\mathrm{E_T}$, and is nearly flat. The systematic uncertainty is evaluated by assuming the largest possible $\mathrm{E_T}$ dependence of this ratio given the available statistics. Comparison of results obtained using a flat ratio to those obtained with a sloped one gives a mass shift of only 0.1 GeV/c^2

The jet $\mathrm{E_T}$ dependence of the SLT tagging efficiency is better known than in the SVX case. However, a systematic uncertainty does arise from the uncertainty in the expected ratio of true to fake SLT tags in $t\bar{t}$ events. To estimate the sensitivity of our top quark mass measurement to this ratio, we generate Monte Carlo $t\bar{t}$ events in which all SLT tags are either (a) true tags, or (b) fake tags. We then produce two large samples of simulated experiments, each having the same number of observed events as in our four mass subsamples and including the appropriate background contributions. The two samples of simulated experiments consist of either events all from set (a) or all from set (b). The median top quark mass values from the two samples of simulated experiments differ by 0.8 GeV/c^2. We take half of this difference, or 0.4 GeV/c^2, as the corresponding systematic uncertainty in the top quark mass. Combining the systematic uncertainties from SVX and SLT tagging, we find an overall systematic uncertainty of ± 0.4 GeV/c^2.

9.5 Parton distribution functions

All of the Monte Carlo samples used to measure the top mass were generated with the MRSD0′ [37] set of parton distribution functions (PDF). This was the preferred PDF at the time the samples were generated. Newer distribution functions now exist, in particular ones which fit CDF's inclusive jet cross section. One such PDF, CTEQ4L [60], provides a higher gluon content at lower momentum fraction than MRSD0′. We take the shift in the median top mass between samples generated with the two PDF's as the relevant uncertainty. We therefore assign a systematic uncertainty of ± 0.3 GeV/c^2 in the top quark mass from this source.

9.6 Monte Carlo generators

The effect of using different Monte Carlo generators has also been studied. Previously, this was evaluated from the difference between the HERWIG and ISAJET simulations. Because of the evidence that independent fragmentation does not reproduce some aspects of the data (energy flow around and between jets [61, 9]) we will not use ISAJET here. We evaluate the systematic uncertainty from the choice of Monte Carlo generators via the mass shift between the HERWIG and PYTHIA simulations. This gives a systematic uncertainty of ± 0.1 GeV/c^2.

9.7 Summary of systematic uncertainties

The relevant systematic uncertainties studied for the top mass measurement are listed in Table 9.1. Combining all of these effects in quadrature gives a total systematic uncertainty of ± 5.3 GeV/c^2, or $\pm 3\%$ of 176.1 GeV/c^2.

Source	Uncertainty (GeV/c^2)
Jet energy measurement	4.4
Initial and final state radiation	2.6
Shape of background spectrum	1.3
b-Tagging	0.4
Parton distribution functions	0.3
Monte Carlo generators	0.1
Total	5.3

Table 9.1: Systematic uncertainties on the measurement of the top quark mass for this analysis.

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