Introduction to the Multivariate Template Method

Outline

- Major features of the Multivariate Template Method (MTM) analysis
- Top mass reconstruction technique
- How we compute the likelihood
- Choices we made for our data measurement
- The top mass result
- Discussion of systematics
- Where we're headed

Analysis Overview

- We measure the top mass in the lepton + jets channel using 1 or 2 SecVtx tags
- A number of features distinguishes MTM from the Run I method
  - A kinematic fit to the W mass includes the jet energy scale (JES), reducing the systematic error
  - Statistical error is reduced by estimating the probability the correct jet-parton assignment in an event was selected
  - Event variables besides the reconstructed mass enter our likelihood function (more info + improved signal/background discrimination)
  - We employ KDE, a non-parametric method of density estimation, in our likelihood calculation

For more info: see CDF Note 6970

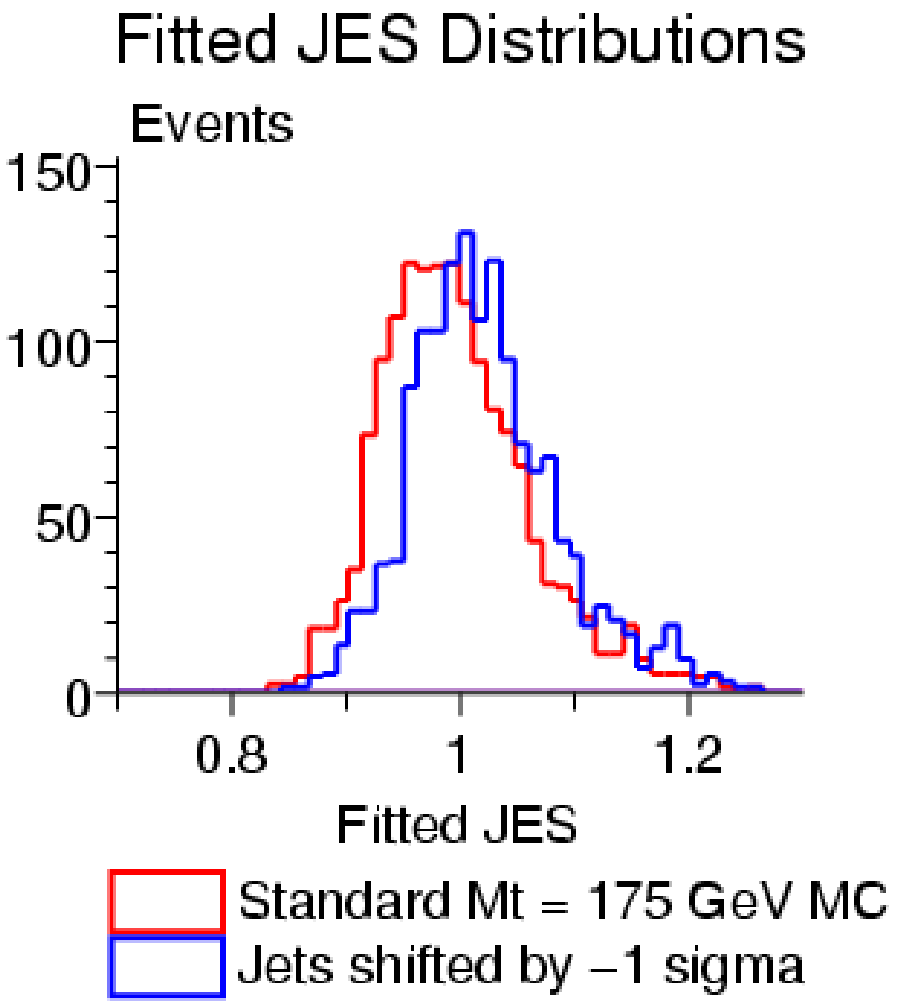
The Data Set

- As we use the cross-section group's background fraction calculations in our analysis, we attempt to use their data set
- Criteria include:
  - 3.5 jets (4th jet > 8 GeV)
  - Wrong beam line runs removed, but wrong luminosity measurement run accepted
  - Trident electrons removed
  - One jet permutation has to agree with the SVX tag information
  - Phoenix electrons non reconstructed and therefore not used in the dilepton veto

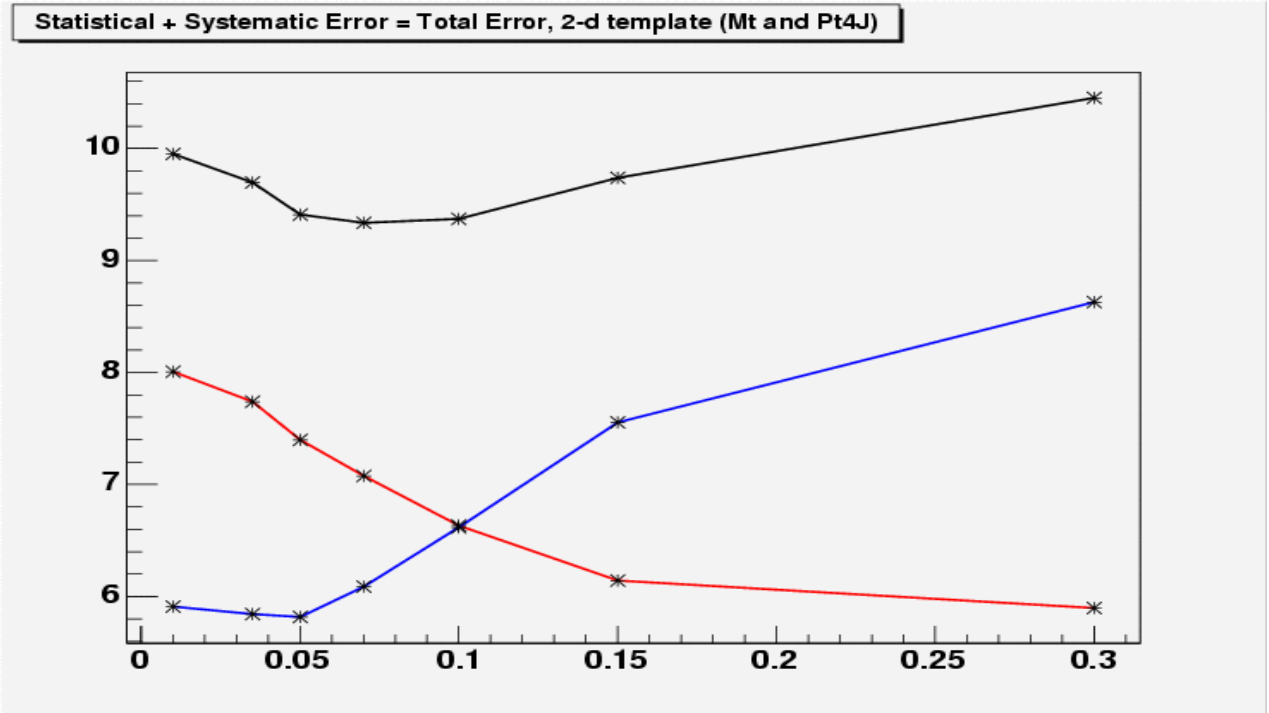
At a JES constraint of 0.07, we end up with 33 events

Mass Reconstruction

- In each event, for every jet-parton permutation we use the JES as a constrained parameter in a kinematic W-mass fit
- This will serve to improve the systematic error, but also increase the statistical error
- By altering the JES constraint, we can alter the resulting tradeoff in errors
- The plot you see indicates that for correctly chosen permutations, the systematic is compensated by the JES shift



The Tradeoff...



As you can see from the above plot of expected pseudo-experiment errors, we can tune the JES constraint to an optimal value. More on this later...

Overview of the Likelihood

$$L(m_t) = \prod_{i=1}^{N} \left(f_b P_b(m_i, x_i) + (1 - f_b) P_s(m_i, x_i, m_t) \right)$$

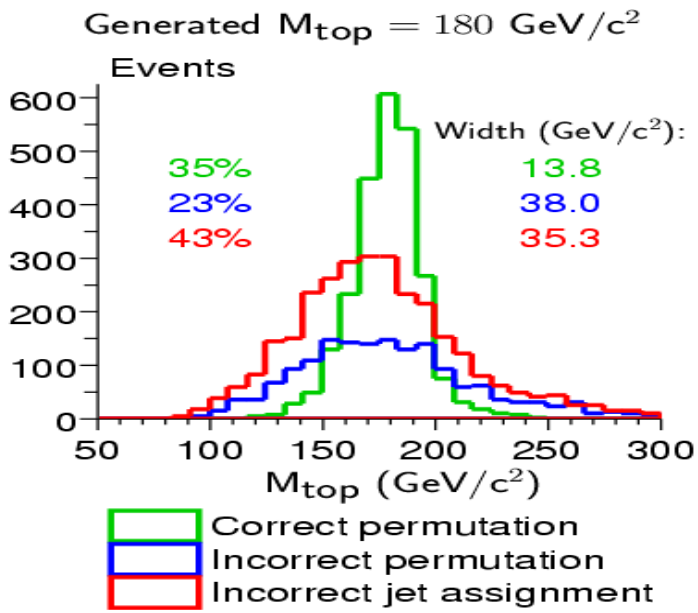
Background: $P_b(m,x) = \sum_{\text{bg types}} a_j B_j(m,x), \qquad \sum_{\text{bg types}} a_j = 1$

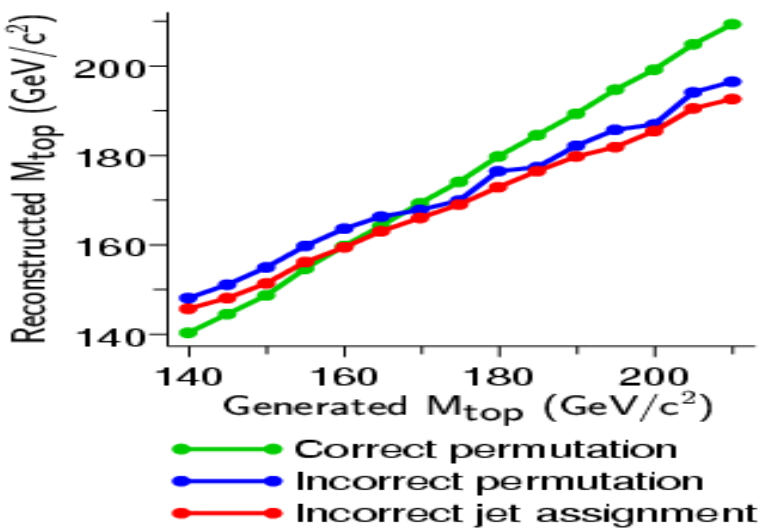
Signal: $P_s(m,x,m_t) = \sum_{\text{sig types}} c_j S_{j,m_t}(m,x), \qquad \sum_{\text{sig types}} c_j = 1$

- Density function templates are assigned to classes of background and signal (B's and S's, respectively)
- These are functions of the reconstructed mass (m) and any other observables we add (x)
- The likelihood of one event is the weighted sum of its probability in each template

Seperate Signal Templates

- The signal is divided into three templates: one for good permutation (GP) events, one for correct jets, bad permutation (BP), and one for incorrect jets (IJ) events
- We've estimated that, assuming negligible background, there could be an improvement in top mass resolution of ~ 1.7 if we perfectly distinguished the signal events





Estimating an Event's Signal Template Probability

- We estimate the probability that an event belongs to one of the signal subsamples in the following steps:
  - First, we take the probability that it belongs to IJ as the ratio of IJ events to all events in our MC files at a given top mass
  - Then, we split the remaining probability into GP and BP probabilities by employing a formula which uses the difference between the best Chisquare and the others :

$$p_{\mathrm{GP}} = \frac{a_b}{\sum_{i=1}^{12} a_i \exp\left(-\frac{\chi_i^2-\chi_b^2}{w_e(\chi_i^2+\chi_b^2)}\right)}$$

...after which a Bayesian update of the GP probability is performed using angle variables in the event

Calculating Background Templates

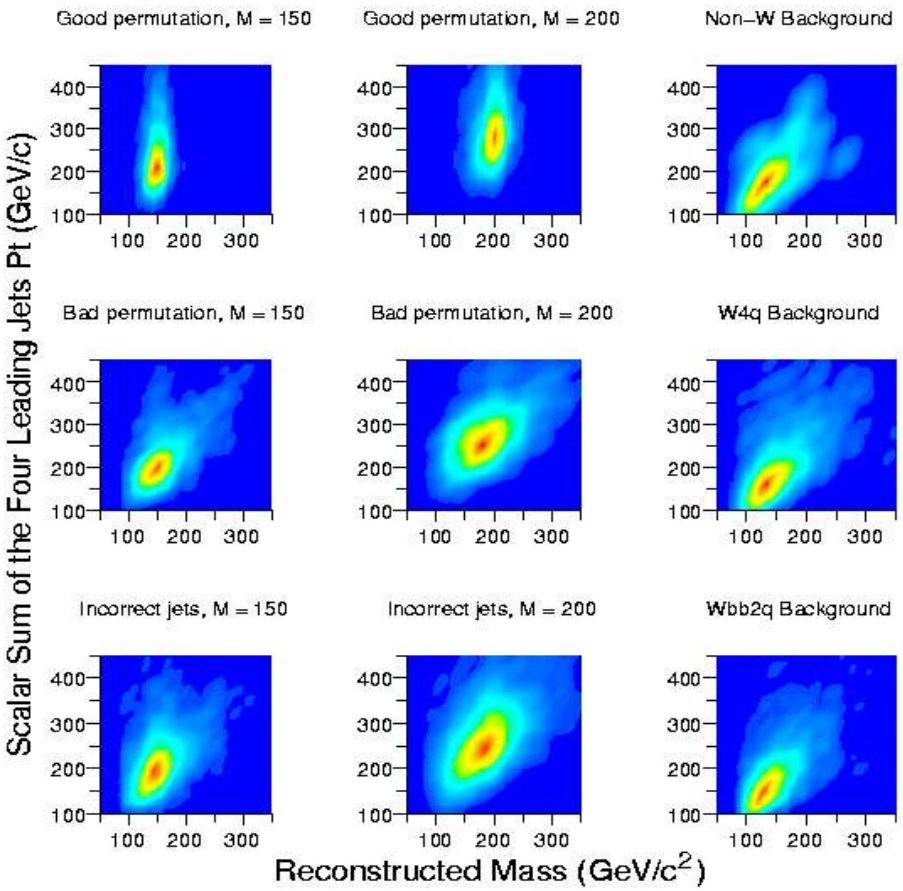
In calculating relative background fractions, we use the cross-section group's values, extrapolating from their 3 jet value to our 3.5 jet value assuming the same 3.5 / 3 jets ratio

For our analysis, we use mistags, QCD, and Wbbar. Wbbar is used for all remaining background types

Background	W+3.5jets	W+≥4jets	Total
mistags	0.90±0.15	0.85±0.13	1.75±0.28
W $b\bar{b}$, W $c\bar{c}$	0.83±0.29	0.60±0.22	1.43±0.51
Wc,WW/WZ etc.	0.41±0.13	0.17±0.07	0.58±0.20
Single top	0.21±0.04	0.09±0.02	0.30±0.06
Sum of above 3	1.45±0.34	0.86±0.24	2.31±0.58
non-W (QCD)	1.24±0.22	1.32±0.25	2.56±0.47
Total Backg	3.59±0.43	3.03±0.37	6.62±0.80
Events Observed	11	29	40
Have a mass fit	10	23	33

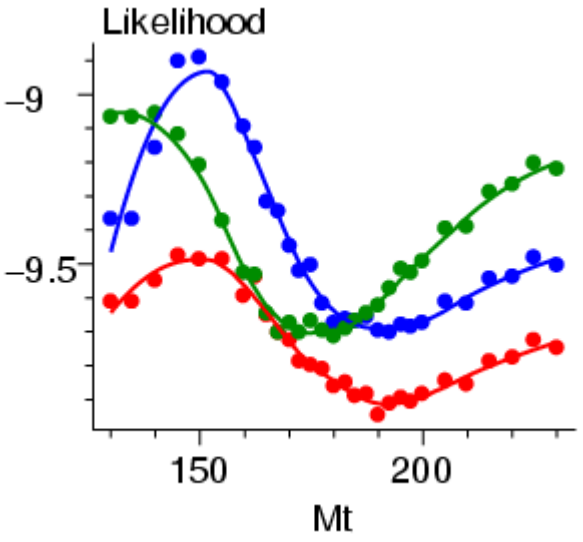
Kernel Density Estimation

- In our likelihood, rather than use a fitted function to represent a continuous density function using a discrete # of MC points, we use KDE
- This technique performs a weighted sum of the surrounding "training points" to estimate the density
- It's advantage is that it frees us from making assumptions as to the form of the density – especially useful in higher dimensions!



Smoothing the Likelihood

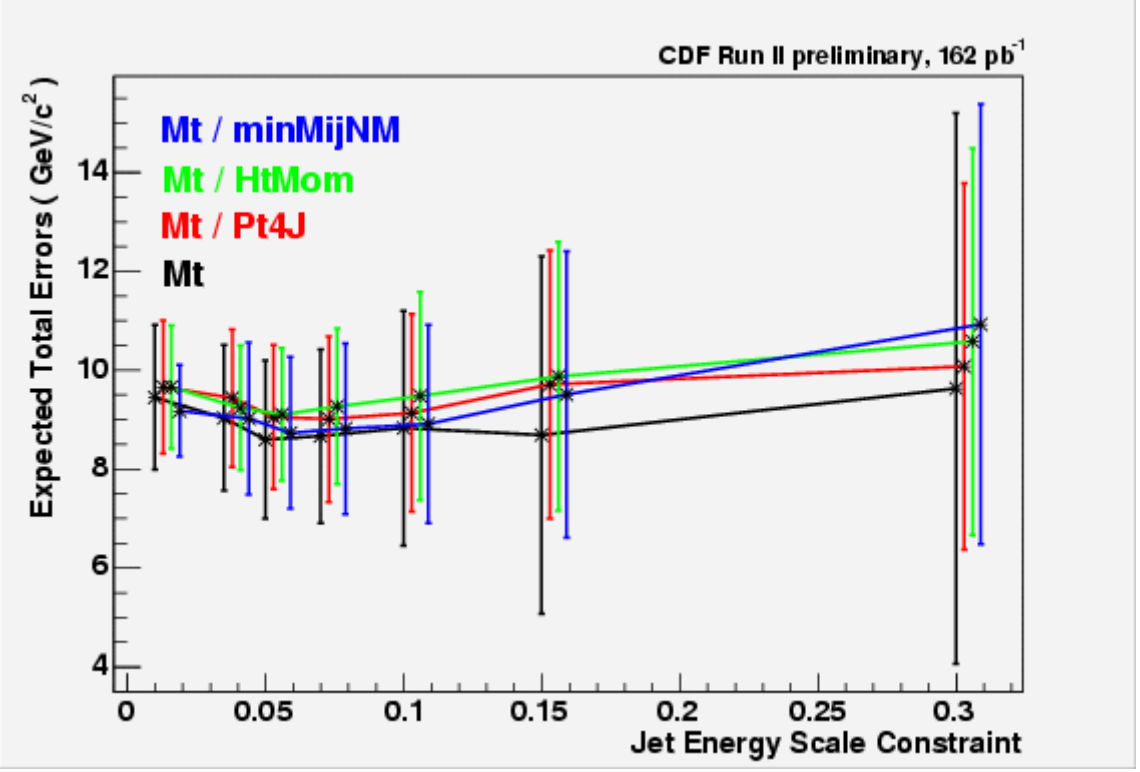
- Since we only have a discrete number of top masses in our signal MC's, it's necessary to interpolate smoothly between the likelihood values of an event
- We do this by employing local polynomial regression
- In the plot, you see two signal and one background event
- For several events, each event is interpolated seperately and then summed



Choices for Our Measurement

- When performing a data measurement on our sample, we need to choose both the JES constraint we'll use and the choice of event variables we'll use in our likelihood function

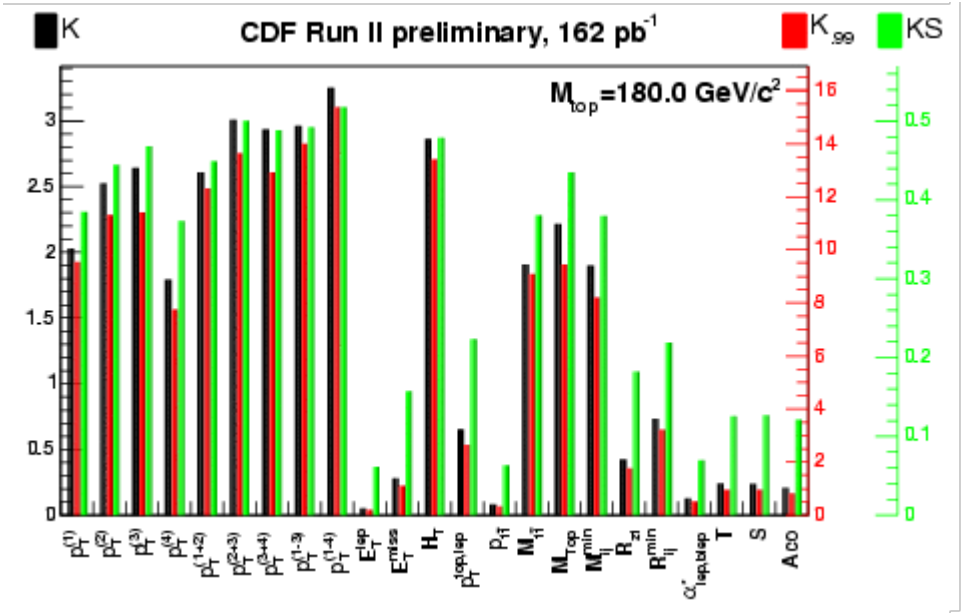
- JES constraint yields lowest error at 0.07
- Relative to the expected error of a single top mass measurement, there's clearly no preferred choice of variables



Choosing a Variable Set

- Using the alpha-skew, Kolmogorov-Smirnov and K-divergence tests, we attempt to determine which variables do the best job at distinguishing signal from background

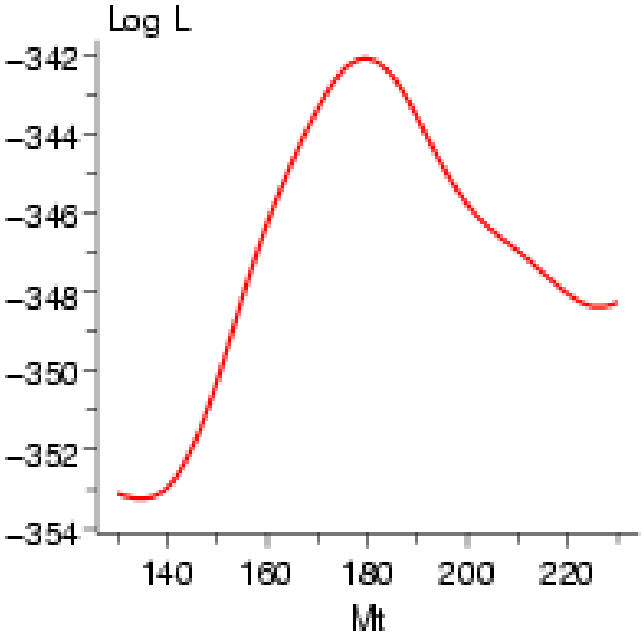
- We find that the scalar sum of the transverse momenta of the leading four jets is the best choice
- For our likelihood, then, we choose the the reconstructed mass and this momentum variable

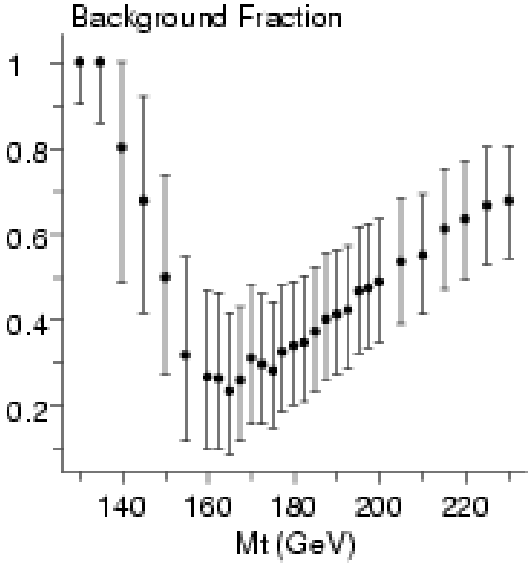


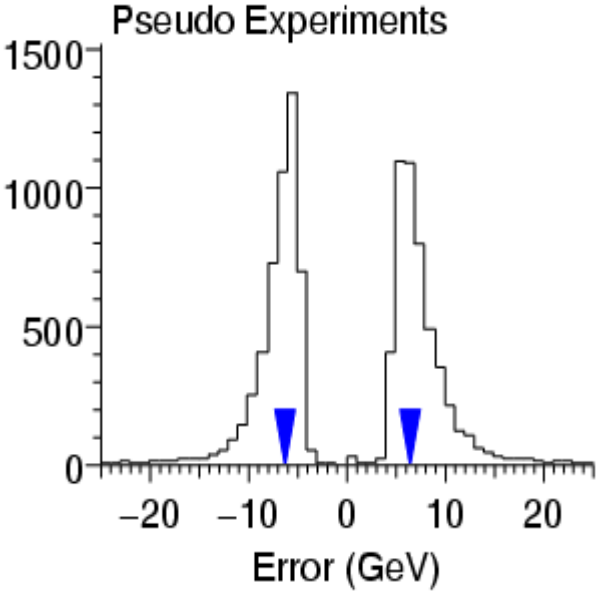
Our Measurement

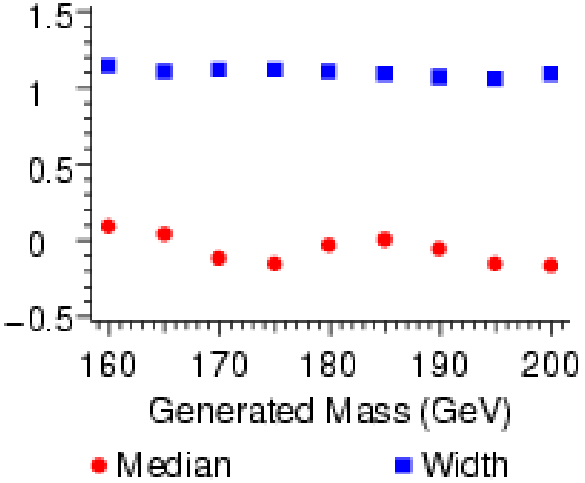
Top Mass: $m_t = 179.6^{+6.4}_{-6.3}$

Estimated background fraction : $f_b = 0.34 \pm 0.14$

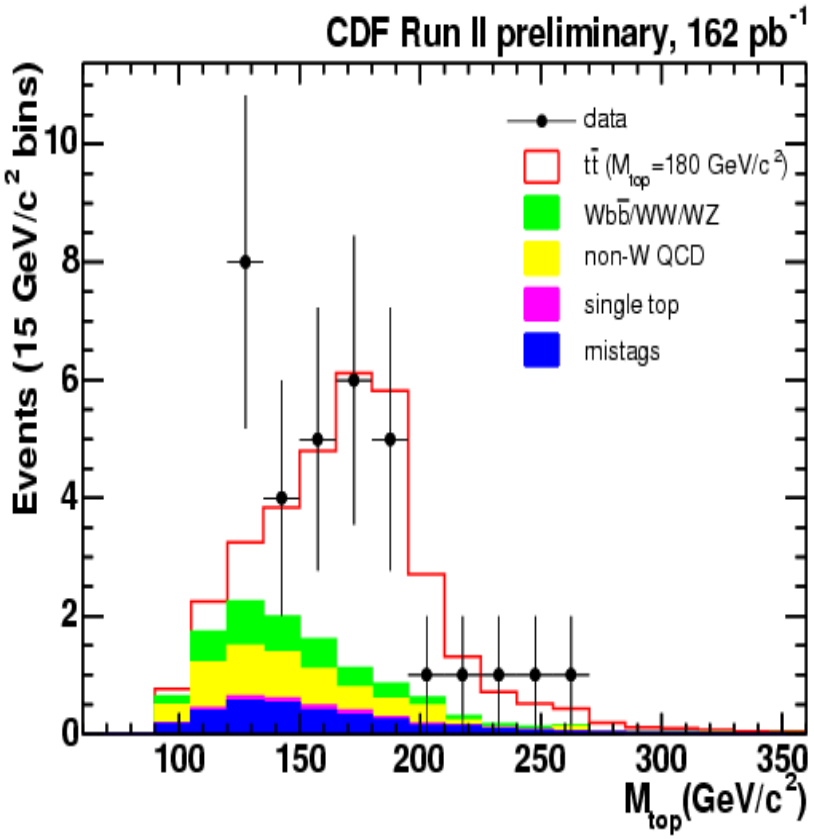


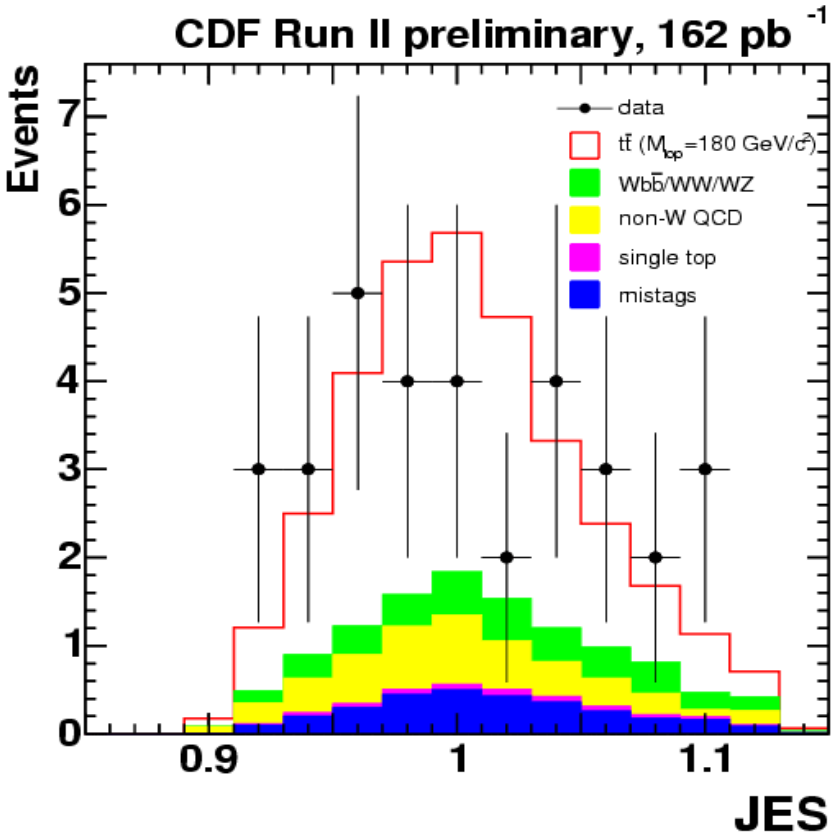






Comparison of Data w/ MC





- Here we see a comparison of our data sample with a normalized combination of signal and background MC. Results are in reasonable agreement.

Systematics

As you can see from this table, the greatest contribution to the systematic error overwhelmingly comes from the jet energy!

Systematic	ΔM_{top} (GeV)
Jet Energy	6.7
Generators	0.2
ISR	0.2
FSR	0.6
PDF	0.6
Background Shape	0.4
b Tagging	0.3
Fitting Procedure	0.7
Total	6.8

Where Our Analysis Is Headed

- Recently, we've tried eliminating events from our sample which have b-jets in the crack region of the detector. From PE's, we've seen the expected total error decrease by almost 1 GeV!
- We've begun an attempt to incorporate transfer functions into our mass reconstruction, to better model the jet errors we expect rather than simply employing a Gaussian. These transfer functions will depend not just on the jet momenta, but also on the # of tracks they contain
- Include other ideas here...

Conclusions

- MTM techniques allow us to find an optimal systematic-statistical error tradeoff
- Our likelihood calculation is improved by estimating probabilities that a given event belongs to a given template
- As our statistics improve, we look forward to re-tuning the JES constraint and reducing our total error

- Currently our blessed top mass measurement is

$$M_{top} = 179.6^{+6.4}_{-6.3} \pm 6.8$$

...but we can improve on this!