Precision Determination of the Top Quark Mass

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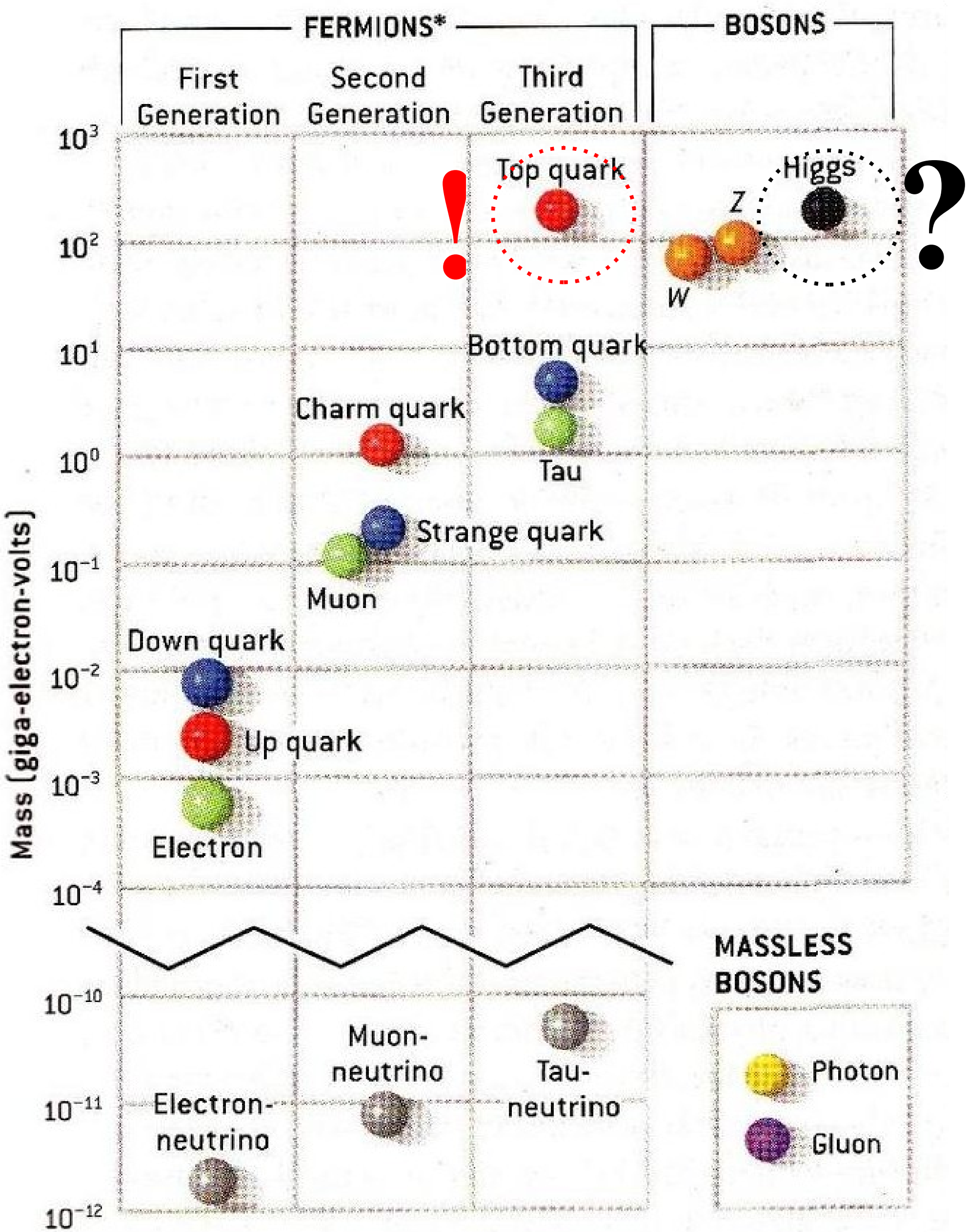
On behalf of the
CDF and DØ Collaborations

The Top Quark in the Standard Model

- Top quark was discovered 1995.
- It is required in the Standard Model (SM) as weak isospin partner of the bottom quark.

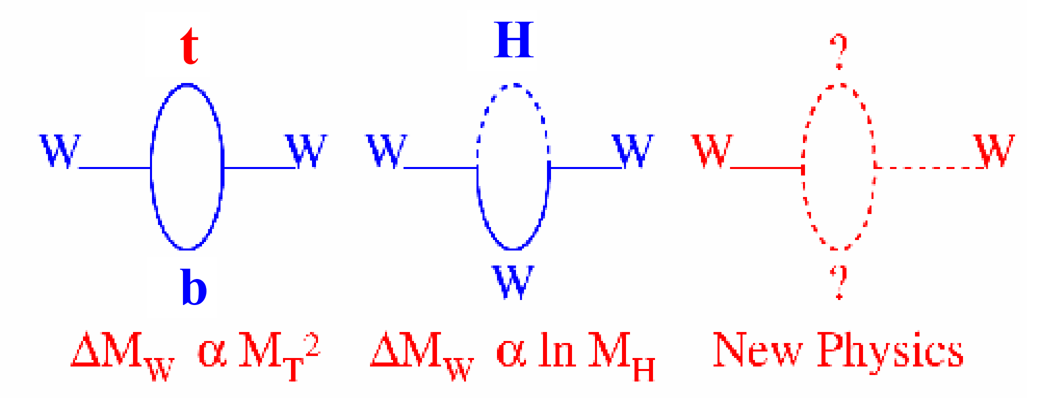
- Striking property: top quark mass is surprisingly large!
  - near electroweak symmetry breaking (EWSB) scale
  - Yukawa coupling ~ 1

- Higgs boson also required by the SM but not seen as yet.



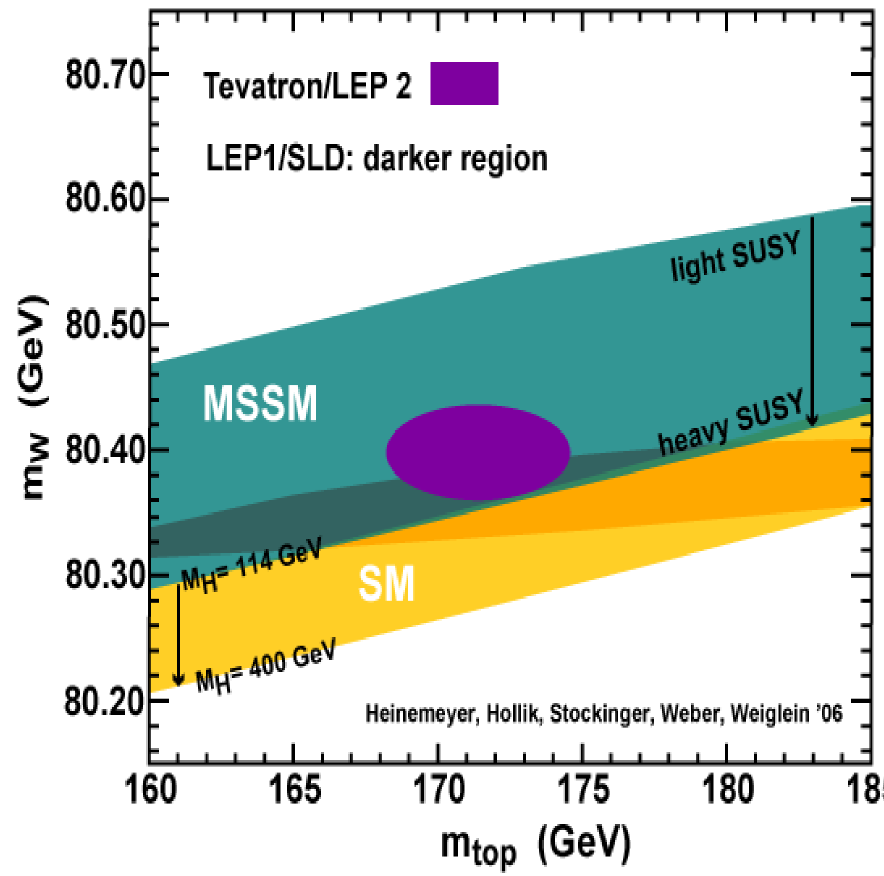
Why Measure the Top Quark Mass?

- Fundamental parameter
- Correlated to other SM parameters via electroweak corrections



- Prediction of the Higgs boson mass.
- Constraints for physics beyond the SM.
- A key to understand EWSB?

Heinemeyer et al. , JHEP 0608:052 (2006)

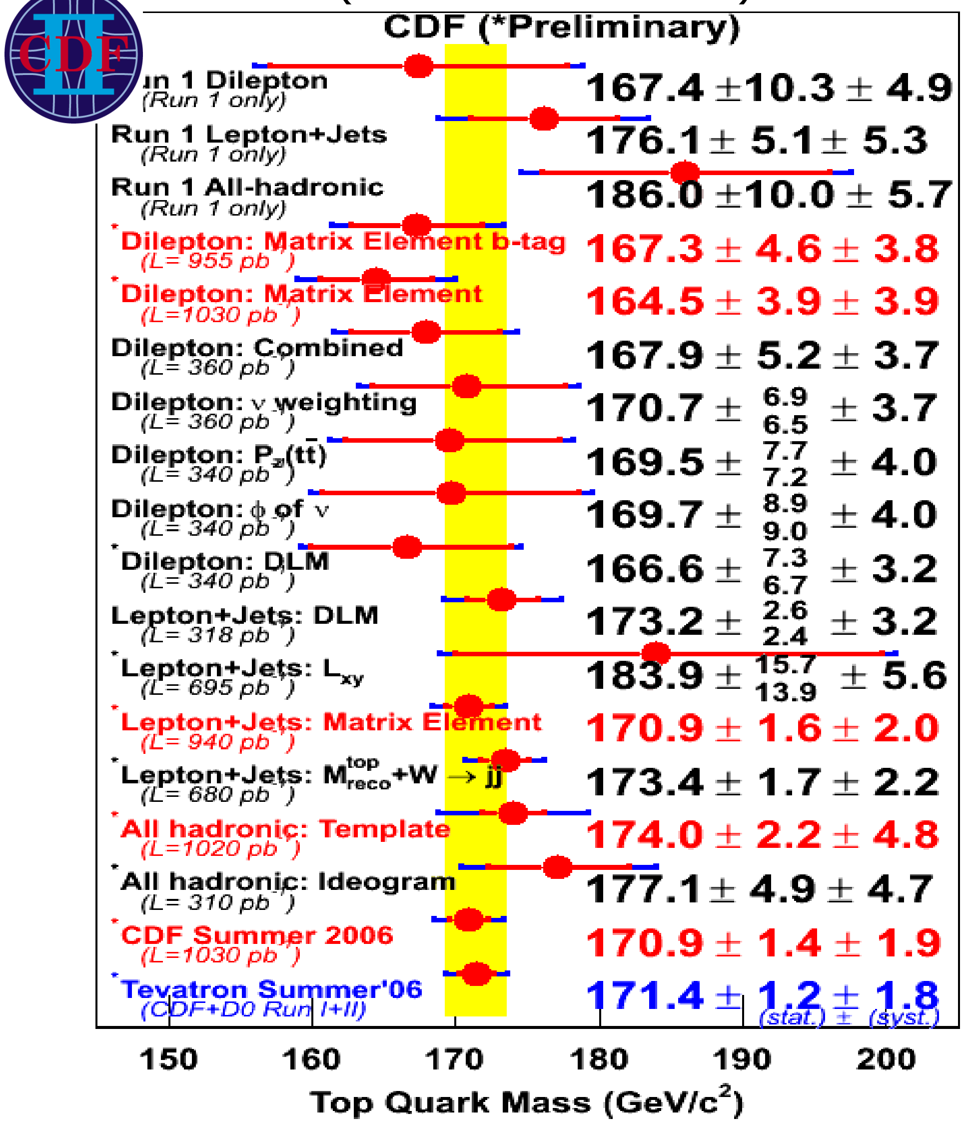


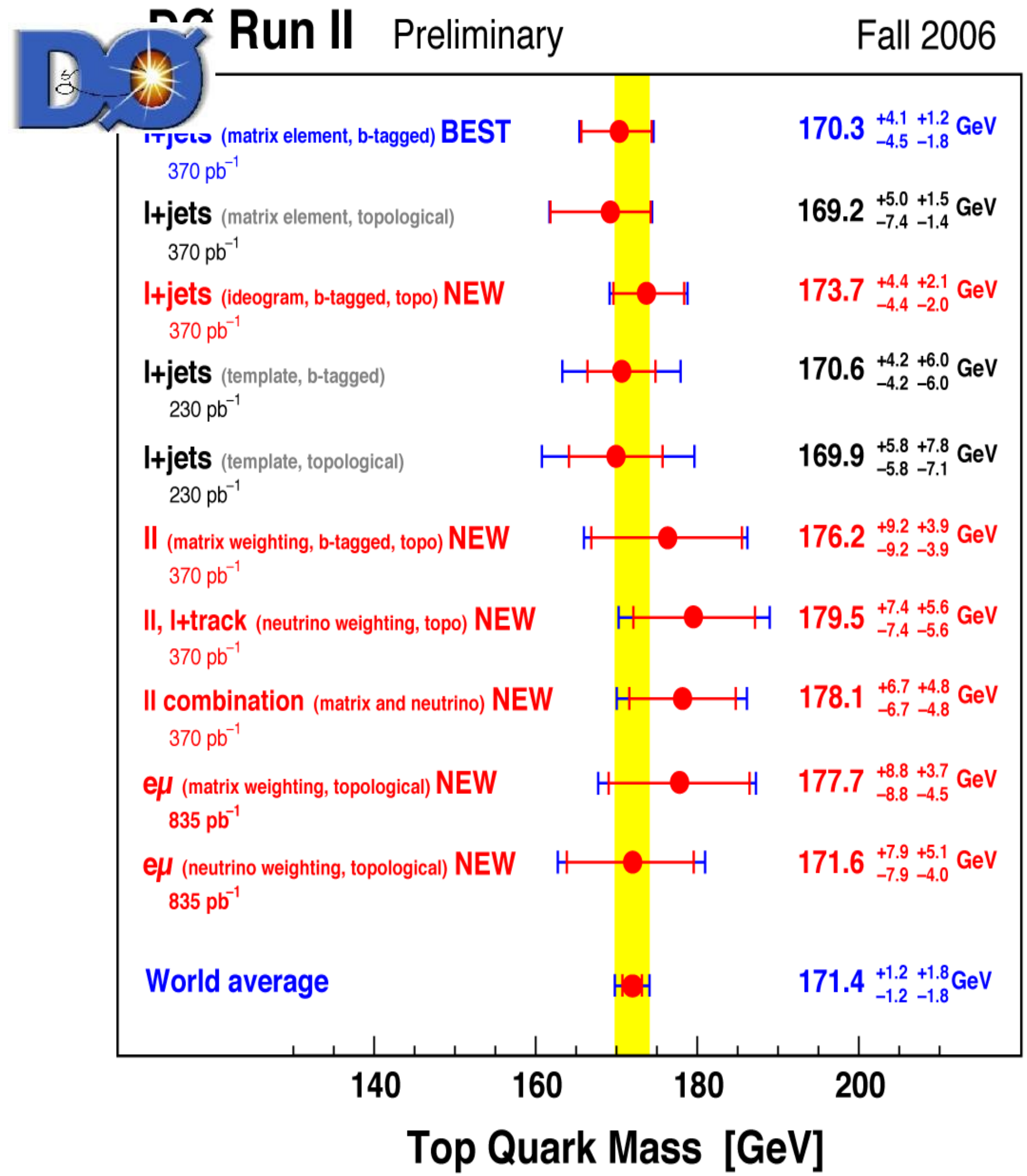
m_{top} and m_W (see C. Hays' talk) currently constrain m_{Higgs} to ~35%!

- Very active field in Tevatron CDF & DØ collaborations with more then 20(!) different measurements competing on the market.

A Snapshot of Tevatron Results

(status summer 2006)

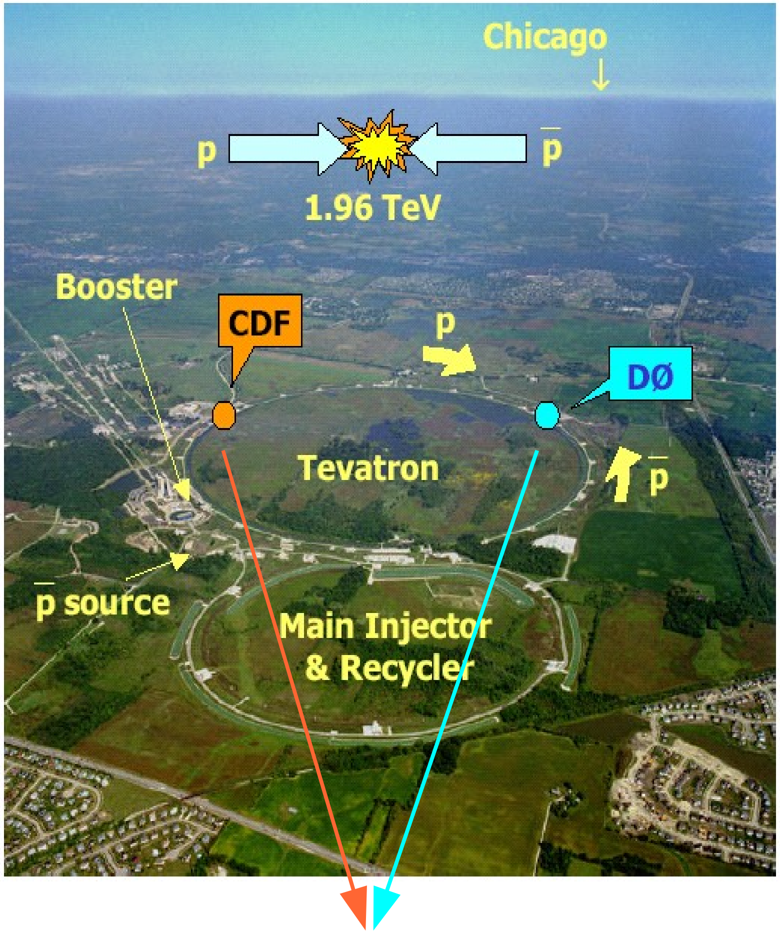




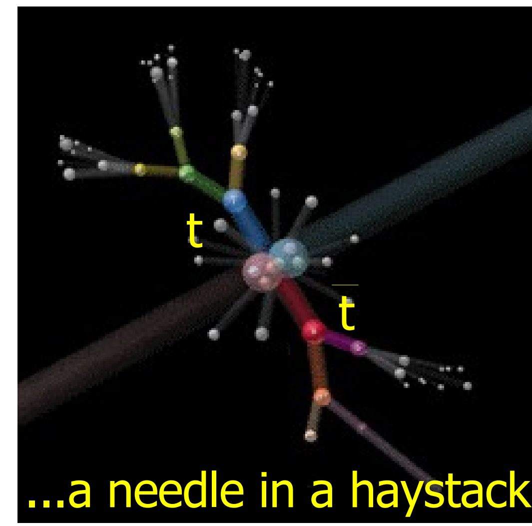
- Impossible to cover them all in one talk ...

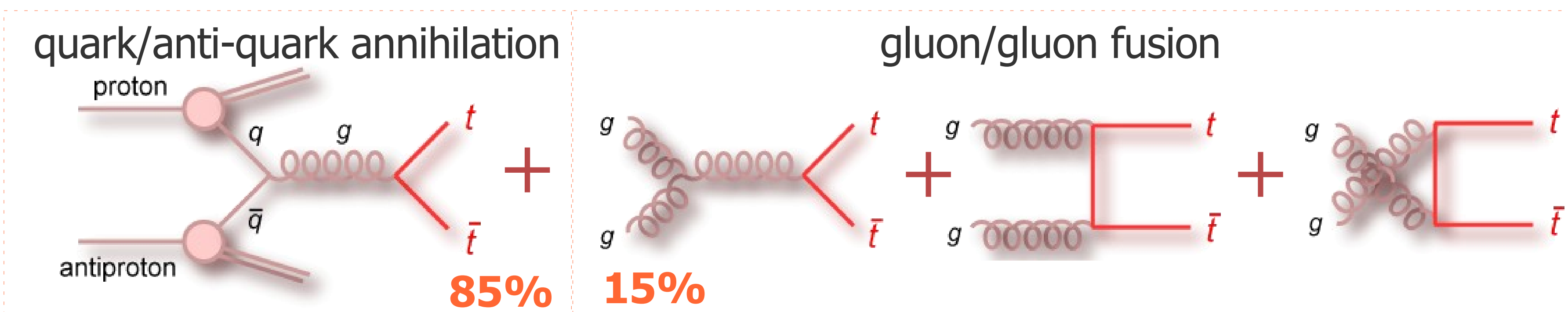
  ...will present the most precise results!

Top Quark Production



- Tevatron is only existing top production machine.
- Run II (since 2001): $\sqrt{s}$=1.96 TeV
- CDF & D0 experiments have ~2/fb on tape. Run-II goal: 4-8/fb.
- Top quarks are mainly produced in pairs via strong interaction: σ_{tt}(1.96TeV)=6.1pb
- 1 top quark pair each 10^{10} inelastic collisions...

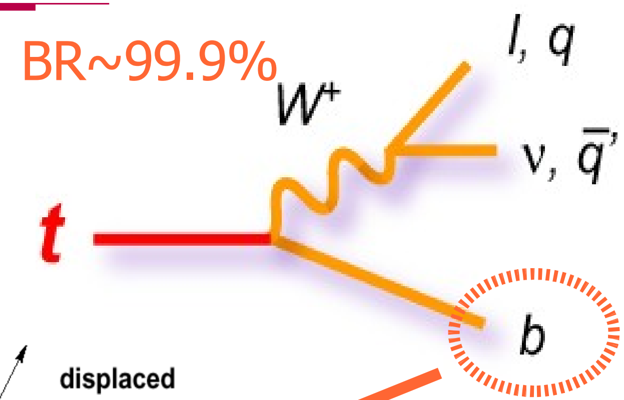




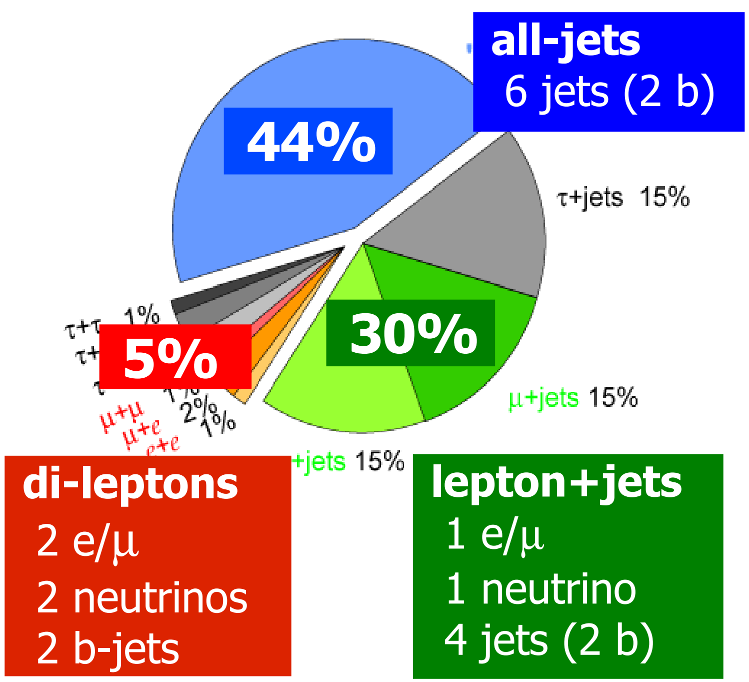
Top Quark Signature

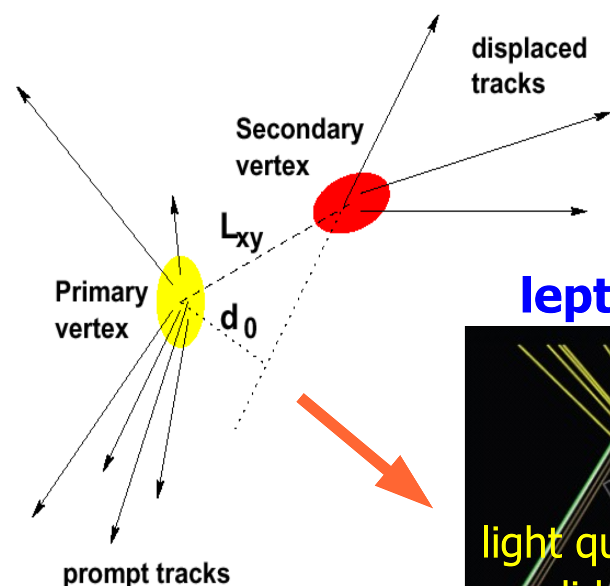
- SM top quark decays weakly before hadronization:
- W decay determines experimental signature:

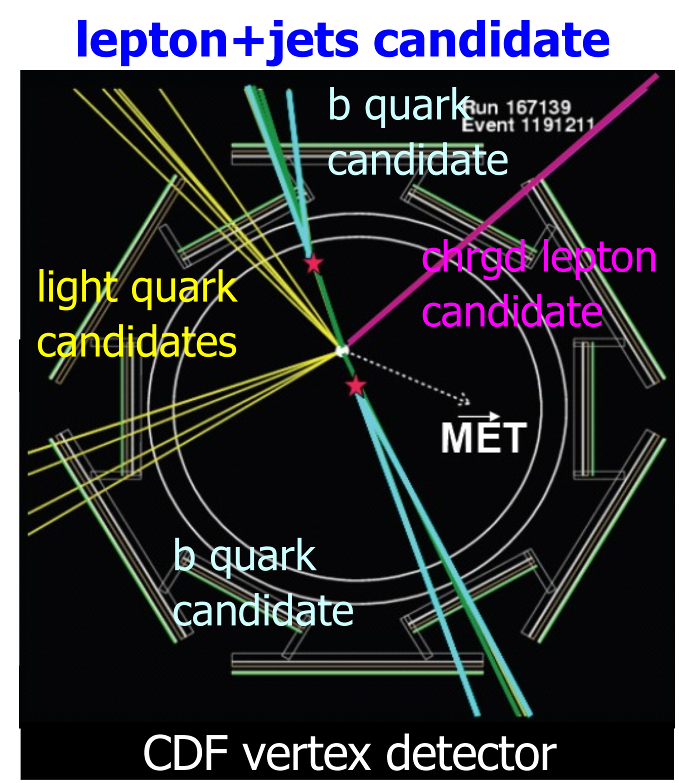
(for more on top properties, see M. Weber's talk.)



Top Pair Branching Fractions





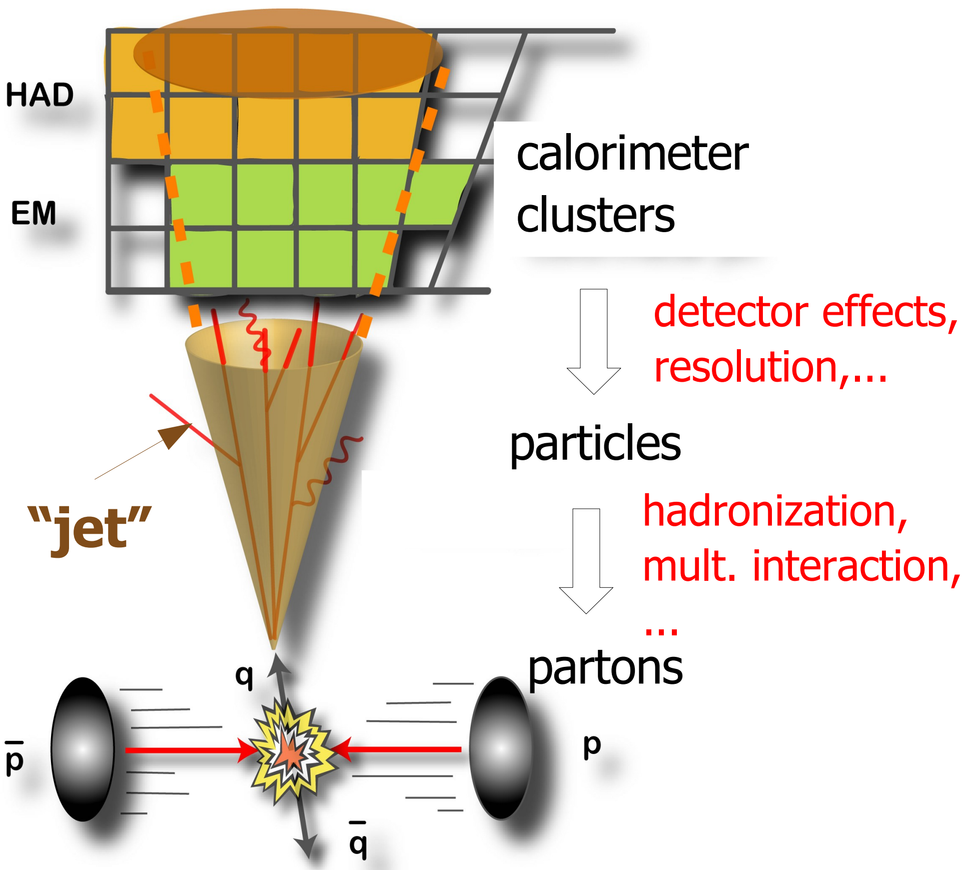


- CDF and DØ have vertex detectors to find displaced vertices from decay of long-lived b-hadrons ...crucial to reduce physics background!

Challenges of Top Quark Physics

- Requires full detector capabilities
  - tracking, calorimetry, hermeticity
  - secondary vertex finding
- Identification of electrons and muons
  → charged leptons from W decay
- Undetected ("missing") energy
  → neutrino reconstruction (p_z unknown)
- Calorimeter clusters ("jets")
  → quark reconstruction
- Secondary vertex tagging
  → quark flavor (b or light)

... reduces physics background and jet/quark combinatorics



Determination of the jet energy scale (JES):

- Correct jet energies for detector effects, hadronization, multiple interactions, ...
  → momenta of hadronic top decay products

JES known to **~3%** → dominant uncertainty in all current top quark mass measurements!

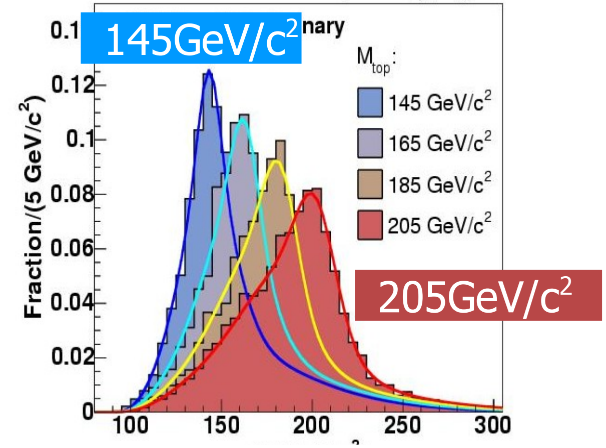
Measurement Strategies

Template Method (TM)

- Calculate a <u>per-event observable</u> correlated with M_{top}.
- Compare simulated distributions (for signal+background) with varying M_{top} with data to obtain M_{top}.
- 2nd variable may be explored for JES determination.

\+ computationally simple

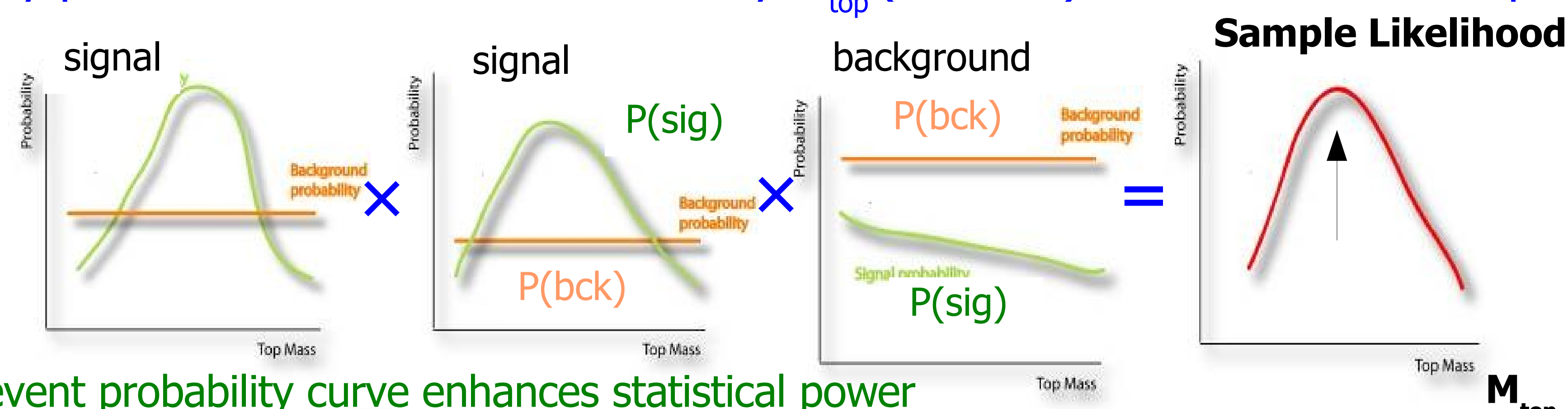
– just <u>one number</u> (for each template variable) per event



e.g.: "reconstructed" top mass

Matrix Element Method (ME)

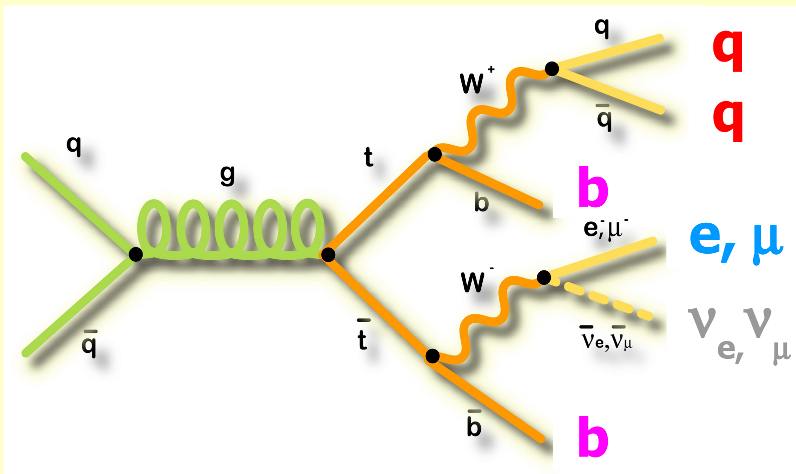
- Calculate a <u>per-event probability density</u> (from ME) for sig.+bkg. as function of M_{top}.
- Multiply probabilities to extract most likely M_{top} (and JES) for whole data sample.

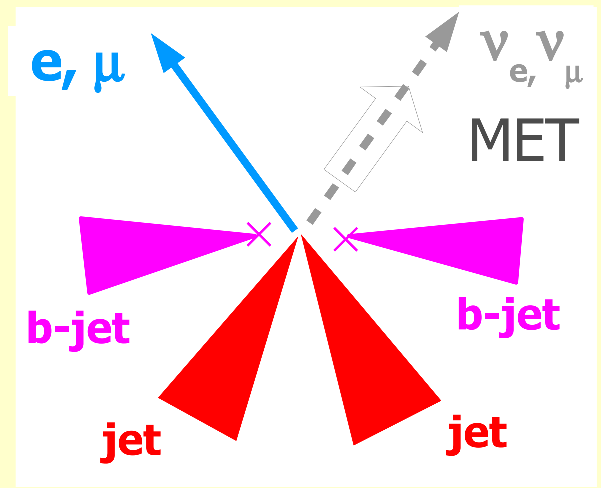


\+ per-event probability <u>curve</u> enhances statistical power

– extremely CPU intensive numerical integration

Lepton-Jets Channel





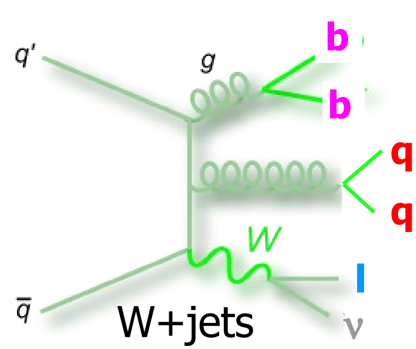
- 1 e/μ with large p_T
- 4 jets with large E_T
- Energy imbalance, high missing E_T
- 0, 1 or 2 b-tags

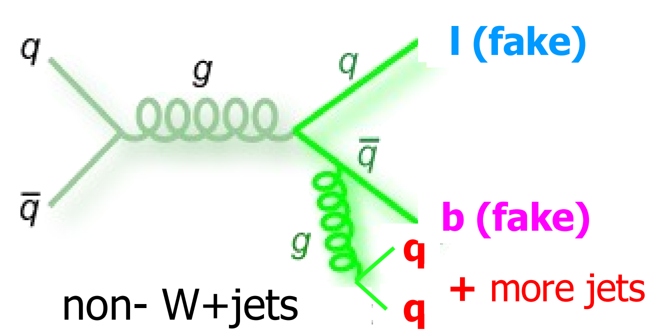
"Golden Channel": Compromise between statistics and purity:

- BR ~ 30%
- S/B=1/4 – 11/1 (depending on b-tag requirement)

- Combinatorial quark/jet ambiguity: 12 (0 b-tag), 6 (1 b-tag), 2 (2 b-tags)
- Well defined kinematics: neutrino momentum partly derived from missing E_T

- Dominant background types:
  - Wbb, Wcc, Wc
  - W+light quarks
  - non-W+light quarks (fake b-tags)





Matrix Element Method

- Maximize mass information by exploring SM predictions for top quark dynamics.

$$P_{t\bar{t}}(M_{top}, \mathrm{JES}) = \frac{1}{N} \sum_{\mathrm{comb}} \int d\sigma_{t\bar{t}}(y, M_{top}) \; dq_1 dq_2 f(q_1) f(q_2) \; w(\boldsymbol{x}, y, \mathrm{JES})$$

sum over all neutrino solutions/ jet-quark combinations

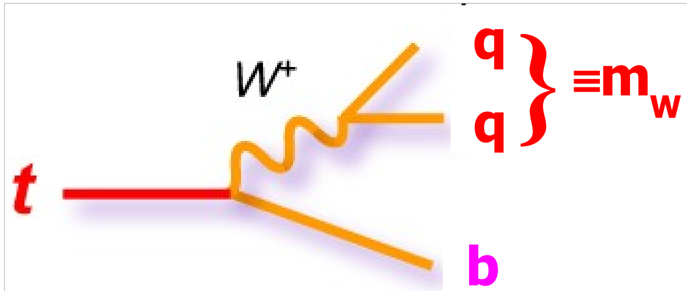
differential cross section
- phase space
- LO tt production ME

proton-parton density functions

"transfer functions" (link jets to quarks)

- Transfer functions are probabilities of a set of variables **x** to be measured given a set of parton level quantities **y**:
  - hadronization and detector resolution effects
  - simplifying assumptions: lepton momenta + jet/lepton angles exactly known

- Similar expression for background probability but no M_{top} dependence.



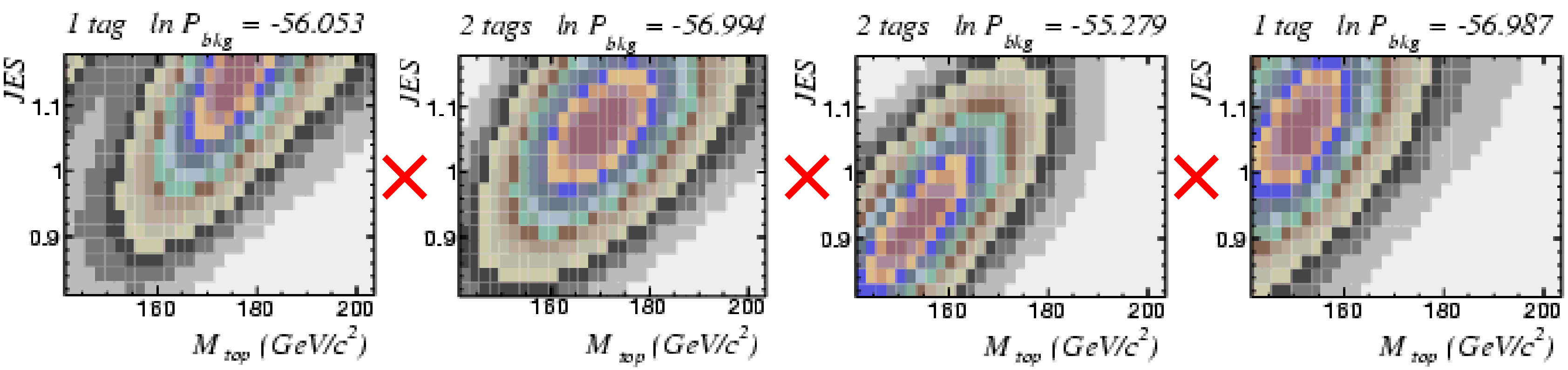
JES is determined "in-situ" using W invariant mass: "Penalty" in probability if JES hypothesis leads to a W mass inconsistent with world average value.

→ Part of JES uncertainty becomes statistical component of top mass uncertainty!

CDF: Matrix Element, Lepton+Jets 955pb^{-1}

sample likelihood: signal probability background probability

$$L(M_{top}, \mathrm{JES}, C_s) \propto \prod_{i=1}^{\mathrm{events}} \left[C_s P_{t\bar{t}}^{(i)}(M_{top}, \mathrm{JES}) + (1 - C_s) P_{\mathrm{bck}}^{(i)}(\mathrm{JES}) \right]$$



- M_{top}, JES extracted in a 2-D maximum likelihood fit

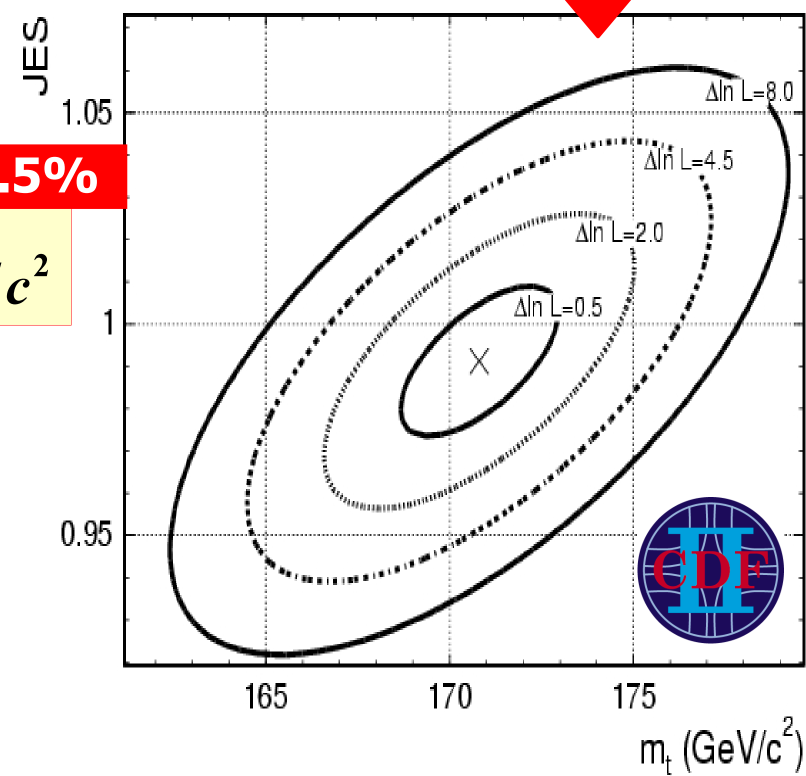
Result using 167 candidate events (≥1 b-tag):

$$M_{top} = 170.8 \pm 1.6(\mathrm{stat.}) \pm 1.5(\mathrm{JES}) \pm 1.4(\mathrm{syst.})\ \mathrm{GeV}/c^2$$

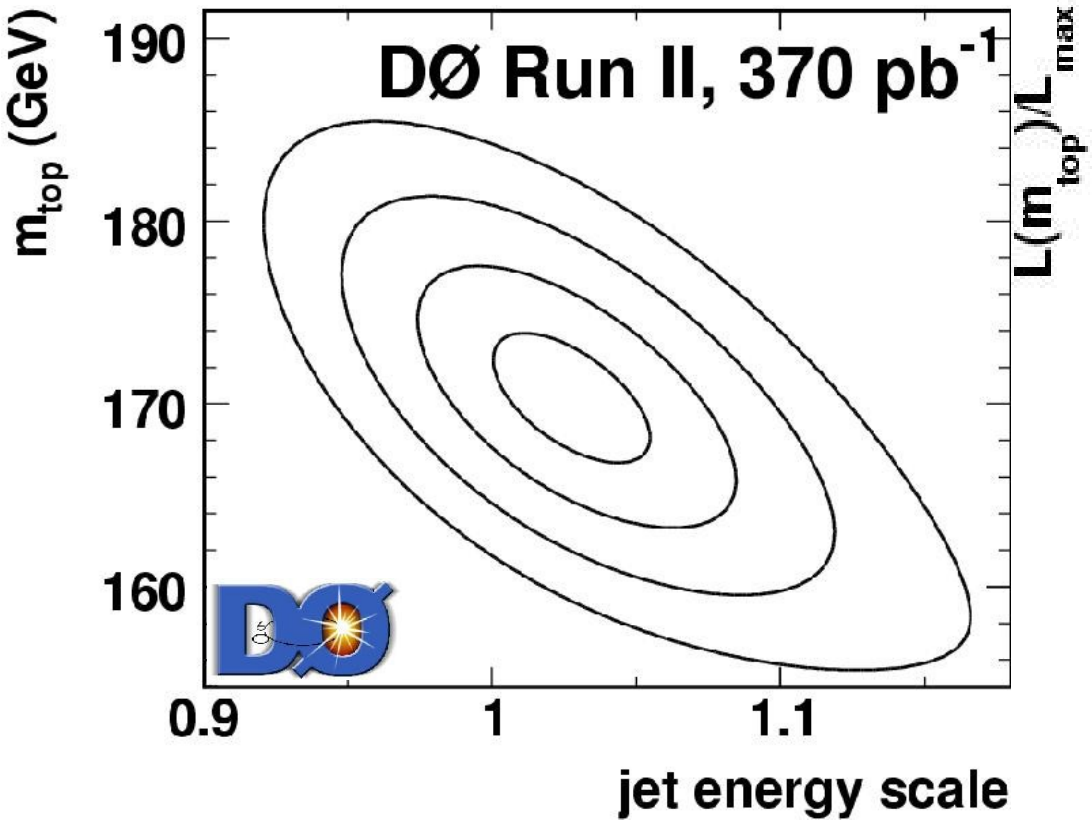
1.5%

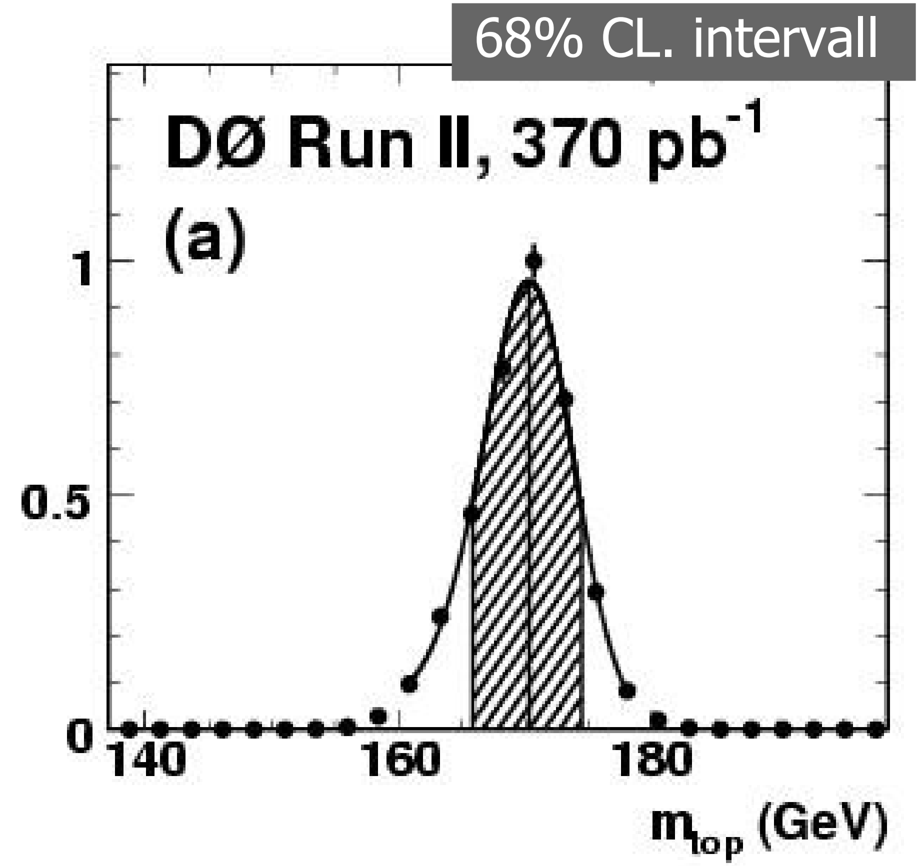
...most precise single top quark mass measurement so far!

- In-situ technique greatly reduces JES uncertainty. Will further scale down with integrated luminosity.



DØ: Matrix Element, Lepton+Jets 370pb^{-1}





- Similar 2-D Likelihood analysis with in-situ JES calibration.
- Includes also events w/o b-tags.

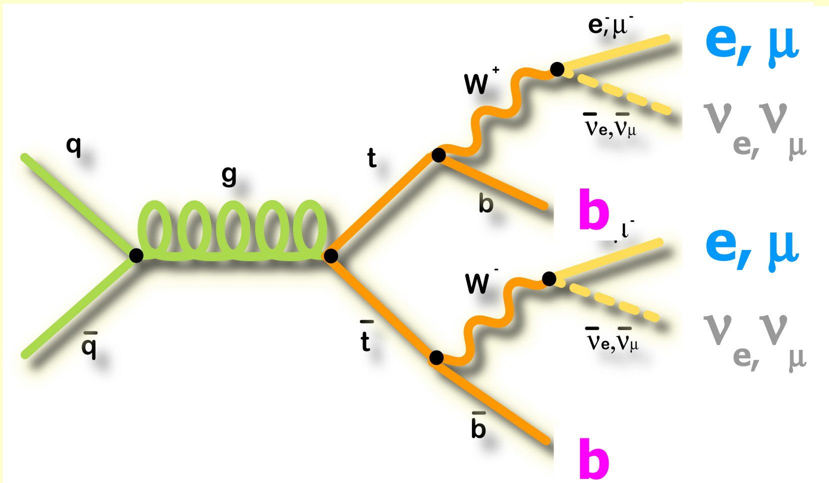
2.6%

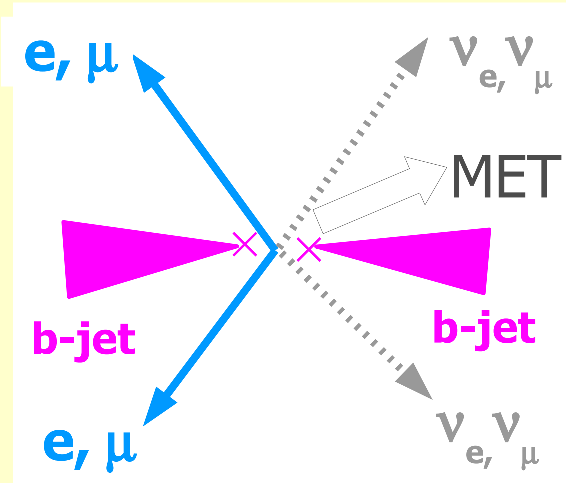
Result using 175 candidate events (≥0 b-tag):

$$M_{top} = 170.3 \pm 2.5\,(\mathrm{stat.}) \pm 3.5\,(\mathrm{JES}) \pm 1.5\,(\mathrm{syst.})\ \mathrm{GeV}/c^2$$

- DØ update coming soon!

Di-Lepton Channel





- 2 opp. charged lepton candidates
- 2 high E_T jets
- ≥0 or ≥1 b-tag
- large missing E_T
- high total transverse energy

Clean sample but poor statistics:

- BR ~ 5%
- S/B ~ 2 (≥0 b-tag)
  S/B ~ 20 (≥1 b-tag)

- Small combinatorial ambiguity: 2 jet-quark assignments
- Under-constrained kinematics: 2 neutrinos but only one missing energy variable
  ...requires assumptions of/integration over unmeasurable quantities to solve M_{top}

- Major background types:
  Z/γ*+2jets WW+2 jets W+3jets (fake leptons)

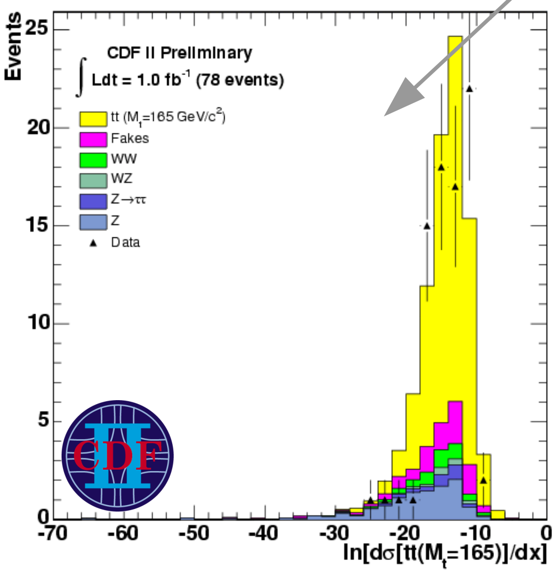
CDF: Matrix Element, Di-Lepton 1030pb^{-1}

- Event probability is weighted sum of signal and of three major backgrounds

$$P_{t\bar{t}}(\boldsymbol{x};M_{\mathrm{top}}) = P_{\mathrm{s}}(\boldsymbol{x};M_{\mathrm{top}})\,w_{\mathrm{s}}(M_{\mathrm{top}}) + \sum_{i=1}^{3} P_{\mathrm{b}}^{(i)}(\boldsymbol{x})\,w_{\mathrm{b}}^{(i)}(M_{\mathrm{top}})$$

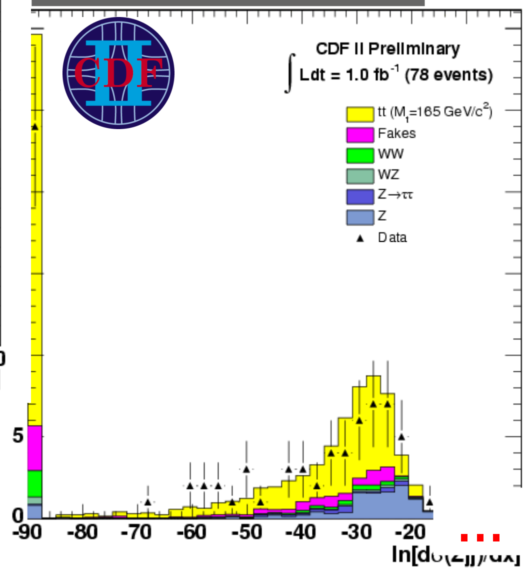
signal from LO matrix element

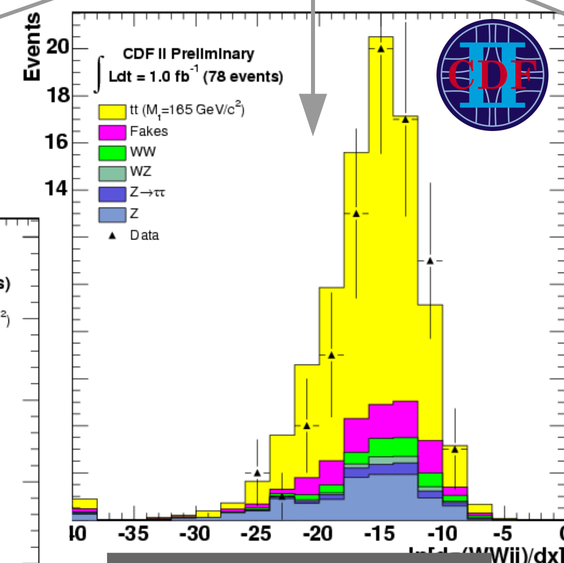
background, fixed weights $w_b^{(i)}$



tt probability distribution

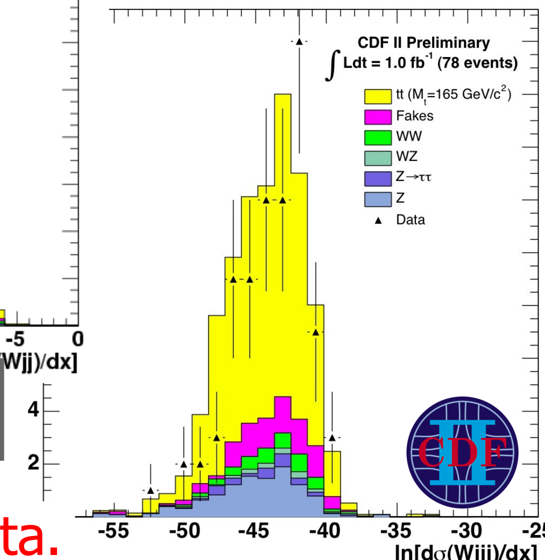
Z/γ qq̄ probability distribution





WWqq̄ probability distribution

fake lepton probability distribution

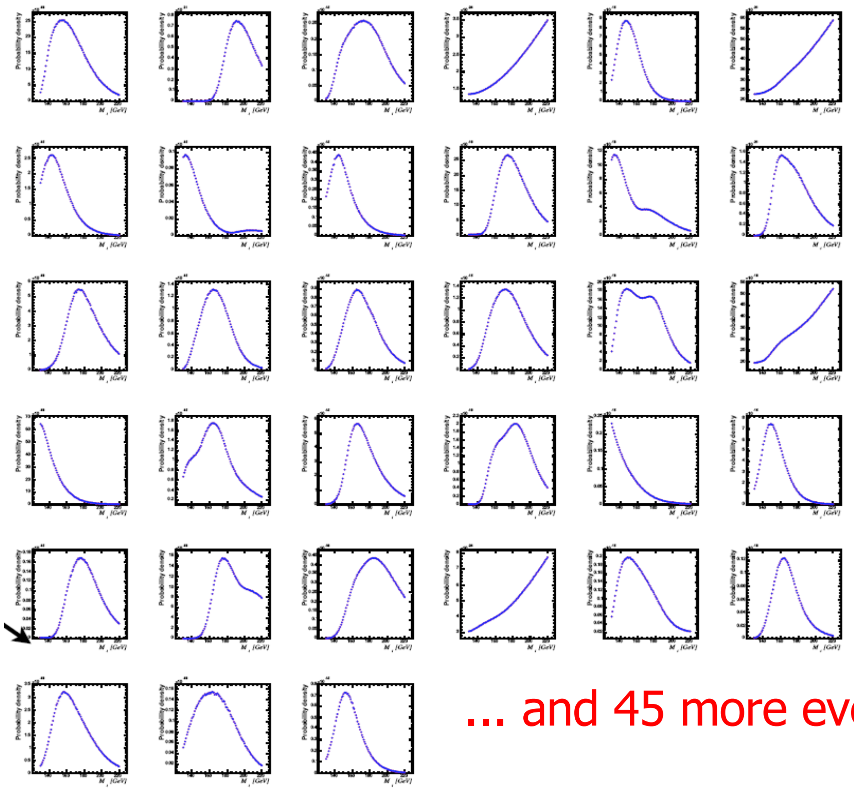


... agree well with data.

Log(dσ/dx)

- Background probabilities reduce M_{top} uncertainty by 15%
- In-situ JES calibration not possible for the signal.

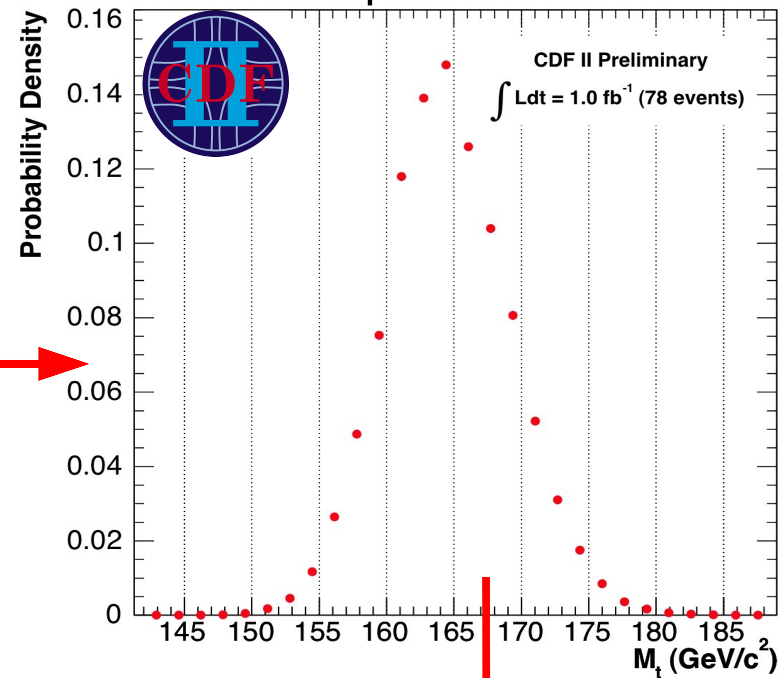
CDF: Matrix Element Method, Di-Lepton, 1030pb^{-1}



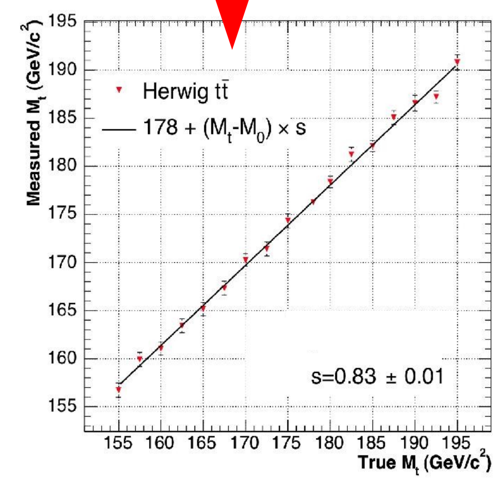
... and 45 more events

$$\prod P_{t\bar{t}}(\boldsymbol{x}; M_{top})$$

sample likelihood



calibrate



slope <1 due to background

3.4%

Result using 78 candidate events (≥0 b-tag):

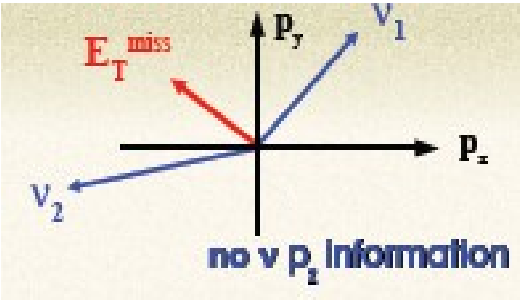
$$M_{top} = 164.5 \pm 3.9\,(\text{stat.}) \pm 3.5\,(\text{JES}) \pm 1.7\,(\text{syst.})\,\text{GeV}/c^2$$

... most precise single di-lepton top quark mass!

Cross check using 30 candidate events (≥1 b-tag):

$$M_{top} = 167.3 \pm 4.6\,(\text{stat.}) \pm 3.3\,(\text{JES}) \pm 1.9\,(\text{syst.})\,\text{GeV}/c^2$$

DØ: Template, Di-Lepton $370pb^{-1}$

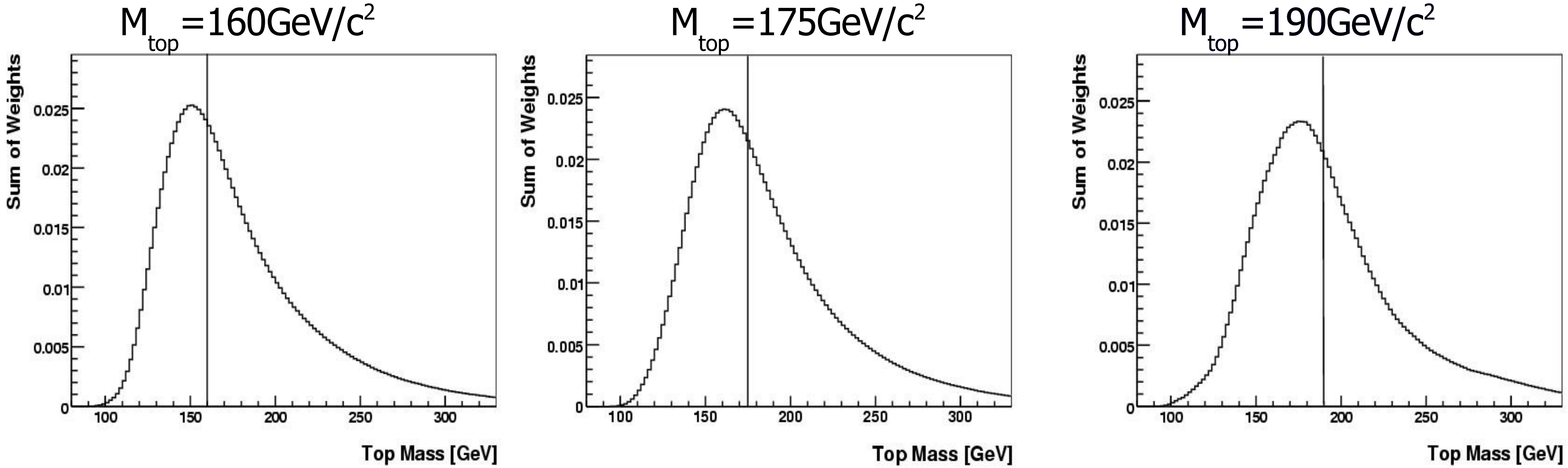


- Di-lepton template methods handle kinematic ambiguity by assuming values for kinematic variables to extract a M_{top} solution and assigning weights to different solutions.

Neutrino Weighting Method: Assume (scan) neutrino pseudo rapidities $\eta(\nu_1)$, $\eta(\nu_2)$ and m_t, assign a weight to the solution based on the compatibility with the observed missing E_T:

$$w(m_t) \propto \sum_{\nu \text{ assumptions}} \exp\left(\frac{-(E_x^{\text{miss, calc}}(i) - E_x^{\text{miss, obs}})^2}{2\sigma^2_{E_x^{\text{miss}}}}\right) \exp\left(\frac{-(E_y^{\text{miss, calc}}(i) - E_y^{\text{miss, obs}})^2}{2\sigma^2_{E_y^{\text{miss}}}}\right)$$

M_{top} templates are formed using sum of weights vs. m_t.



DØ: Template, Di-Lepton 370pb^{-1}

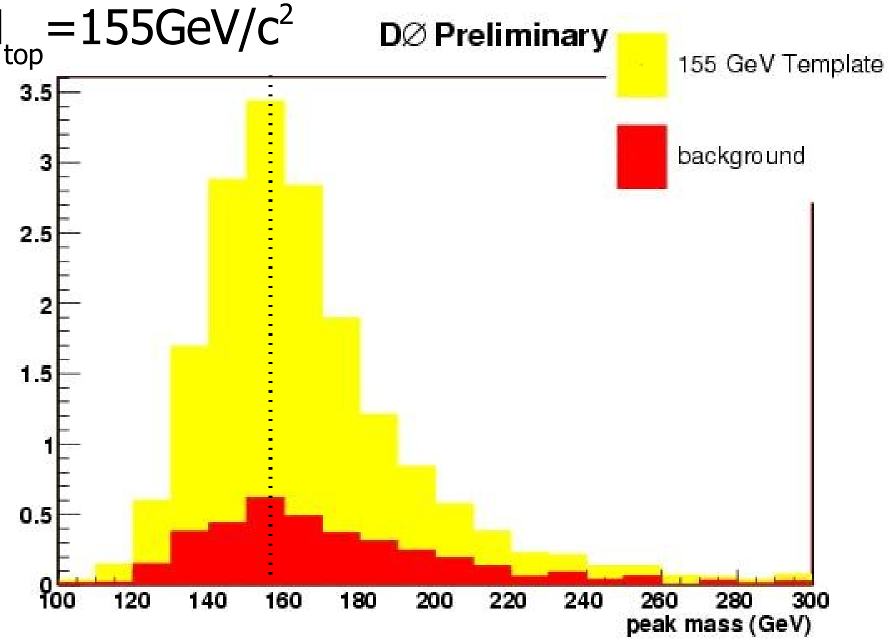
Matrix Element Weighting Method: Assume (scan) over m_t and at most 4 ν solutions (given a m_t, m_W, lepton/quark/missing E_T configuration), assign a weight based on the compatibility of ME prediction with the observed lepton transverse momenta:

$$w(m_t) \propto \sum_{\nu \text{solutions}} \sum_{\text{jets}} f_{\text{PDF}}(x_{q_1}) f_{\text{PDF}}(x_{q_2}) p(E_l^*; m_t) p(E_{\bar{l}}^*; m_t)$$

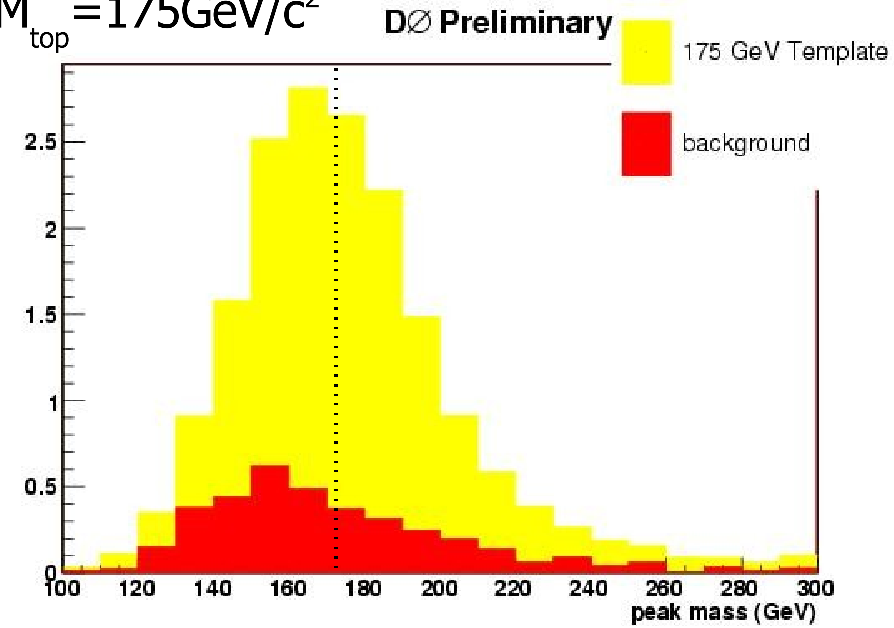
M_{top} templates are formed using m_t values which gives maximum weight ("peak mass").

- Repeat calculations with jet/lepton momenta/missing E_T randomly smeared within their detector resolutions, solve the equations and average the weights.

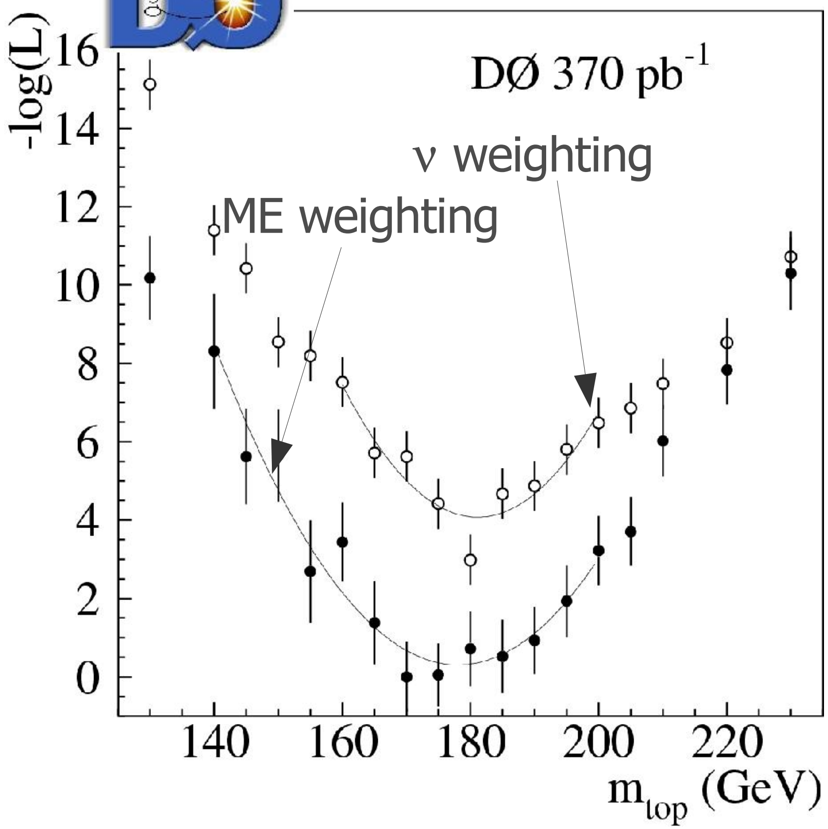
M_{top}=155GeV/c^2



M_{top}=175GeV/c^2



DØ: Template, Di-Lepton $370 pb^{-1}$



- M_{top} obtained from max. likelihood fit for ν-weighting and binned likelihood fit for ME weighting.
- Both results are combined considering correlations.

4.7%

Result using 26⊕36 candidate events (**370/pb**):

$$M_{top} = 178.1 \pm 6.7(\text{stat.}) \pm 4.3(\text{JES}) \pm 2.1(\text{syst.})\,\text{GeV}/c^2$$

5.3%

Prel. result using 28 candidate events (**835/pb**):

$$M_{top} = 171.6 \pm 7.9(\text{stat.})\,^{+5.1}_{-4.0}(\text{syst.})\,\text{GeV}/c^2$$

(eμ channel only)

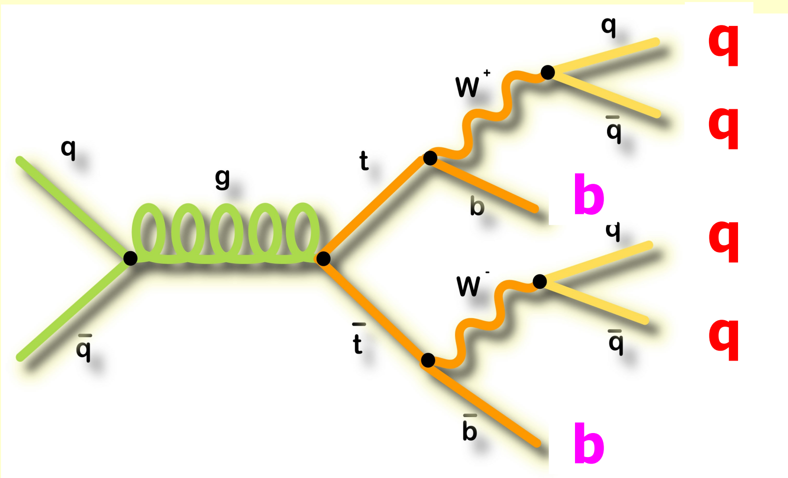
- **New**: CDF template di-lepton analysis based on 1/fb data set:
- Makes assumptions about the longitudinal momentum of the $t\bar{t}$ system to solve equations (see appendix):

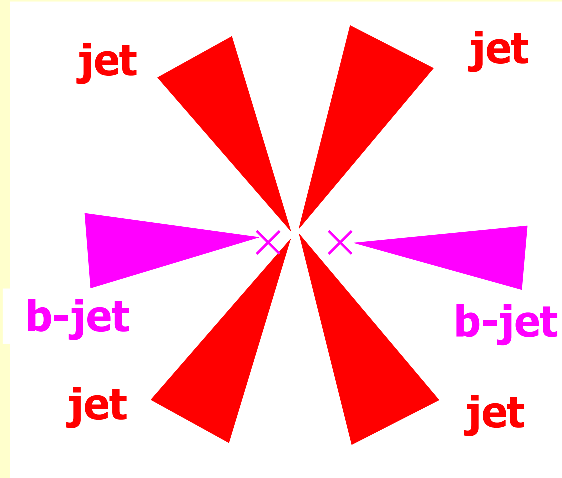
4.1%

Result using 64 candidate events (**1030/pb**)

$$M_{top} = 168.1\,^{+5.6}_{-5.5}(\text{stat.}) \pm 3.2(\text{JES}) \pm 2.4(\text{syst.})\,\text{GeV}/c^2$$

All-Jets Channel





Good statistics but huge background:

- BR ~ 44%
- S/B ~ 1/23 (≥0 b-tag)
  S/B ~ 1/6 (≥1 b-tag)

- Exactly 6 jets with high E_T
- Lepton veto
- Low missing E_T significance
- ≥1 or 2 b-tags
- Large total transverse energy
- Spherical isotropic event topology

- Large combinatorial ambiguity: 90 (1 b-tag), 24 (2 b-tags)
- Well measurable kinematics, no neutrinos.

- Dominant background types:
  non-W bb4q non-W 6q (fake b-tags)
- Additional signal probability cut (from ME calculation) yields
  S/B ~ 1/1 ... very restrictive but usable for >1/fb.

CDF: Template Method, All-Jets, 943pb^{-1}

- 2-D templates for M_{top} and JES: Signal from ME, background model from data. (0 b-tag sample, has negligible signal)

- Signal+background probability densities:

P (m_t | M_{top}, JES) P (m_W | M_{top}, JES)

$$L_{1,2\ \text{b-tag}} = L^{(\text{top})}_{\text{shape}} \times L^{(W)}_{\text{shape}} \times L_{\text{obs}} \times L_{\text{sig}}$$

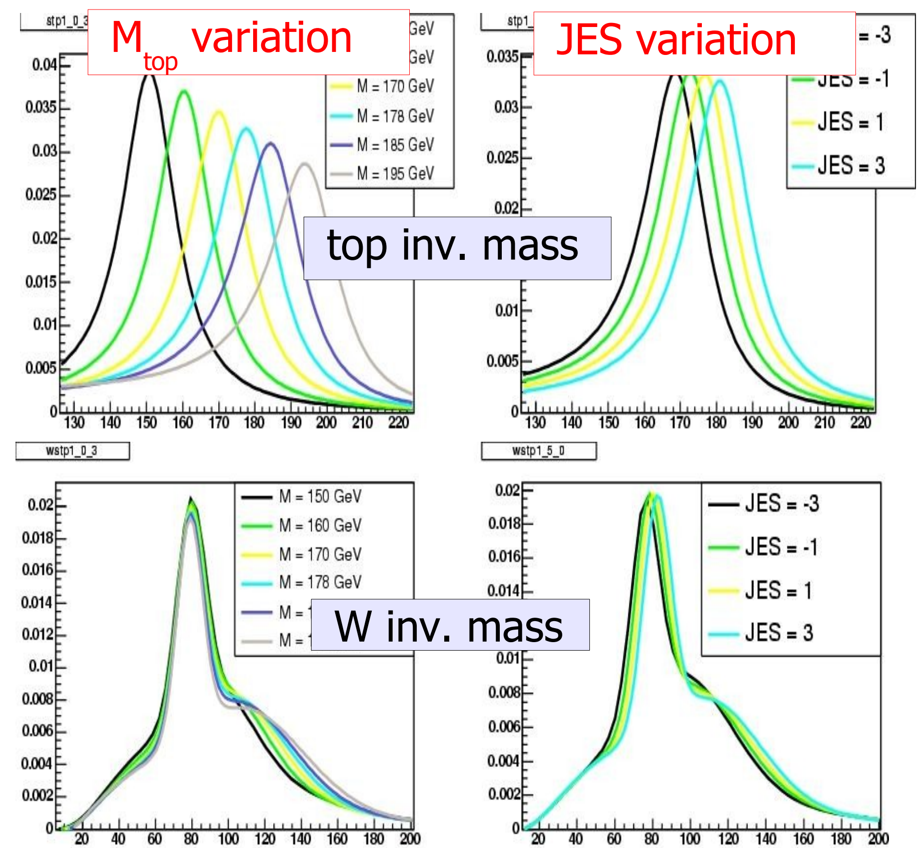
constrain to number of observed events

constrain to number of signal events

- Sample likelihood:

constrain to a priori JES

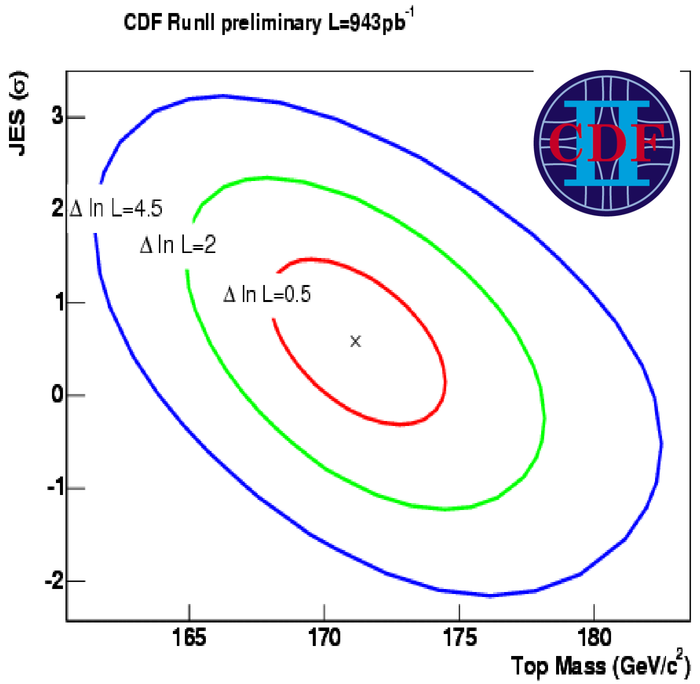
$$L = L_{1\ \text{b-tag}} \times L_{2\ \text{b-tags}} \times L_{\text{JES}}$$

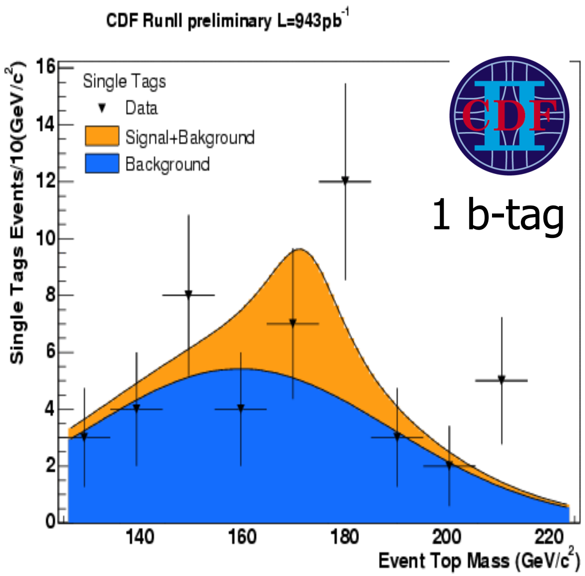


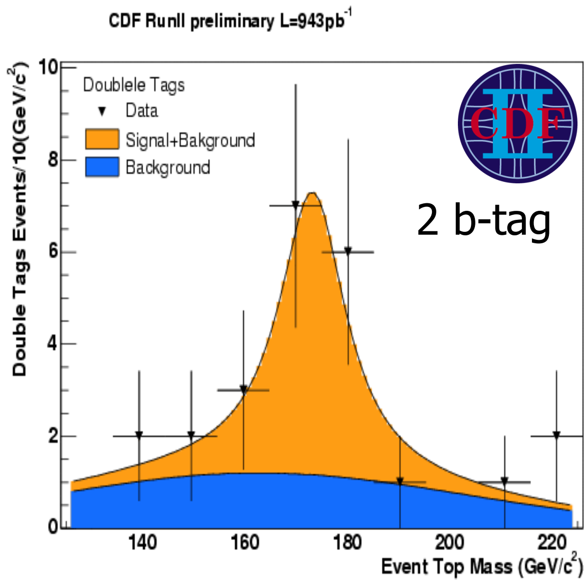
- Likelihood is maximized w.r.t:

M_{top}, JES & number of 1(2) b-tagged signal/back. events respecting constraints (background fraction poorly known in All-Jets channel!)

CDF Template Method, All-Jets, 943pb^{-1}







2.5%

Result using 64 candidate events (≥ 1 b-tag):

$$M_{top} = 171.1 \pm 2.8\,(\text{stat.}) \pm 2.4\,(\text{JES}) \pm 2.1\,(\text{syst.})\,\text{GeV}/c^2$$

- First All-Jets result with in-situ JES.
- All-Jets channel becomes competitive!

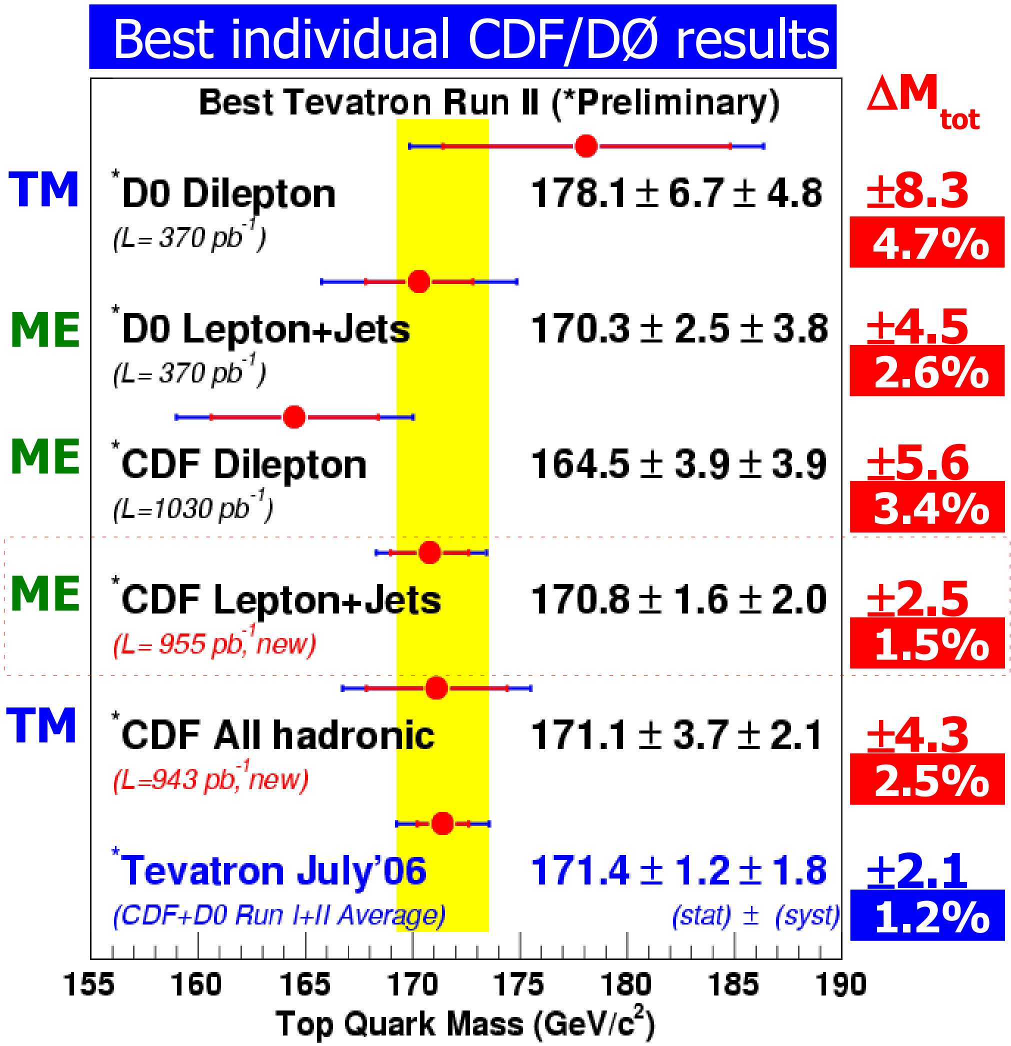
- Recent result from “traditional” 1-D template method using a kinematic mass fitter:
  - no in-situ JES calibration, no restrictive signal probability cut:

3.0%

1-D template, 1020pb^{-1}, 772 candidate events (≥ 1 b-tag):

$$M_{top} = 174.0 \pm 2.2\,(\text{stat.}) \pm 4.5\,(\text{JES}) \pm 1.7\,(\text{syst.})\,\text{GeV}/c^2$$

Comparisons



(status 03/07/2007)

- Combination of best Run-I & II results for each experiment (new CDF All-Jets result not included here):

$$M_{top}(\text{all-jets}) = 173.4 \pm 4.3\,\text{GeV}/c^2$$
$$M_{top}(\text{lep-jets}) = 171.3 \pm 2.2\,\text{GeV}/c^2$$
$$M_{top}(\text{di-lepton}) = 167.0 \pm 4.3\,\text{GeV}/c^2$$

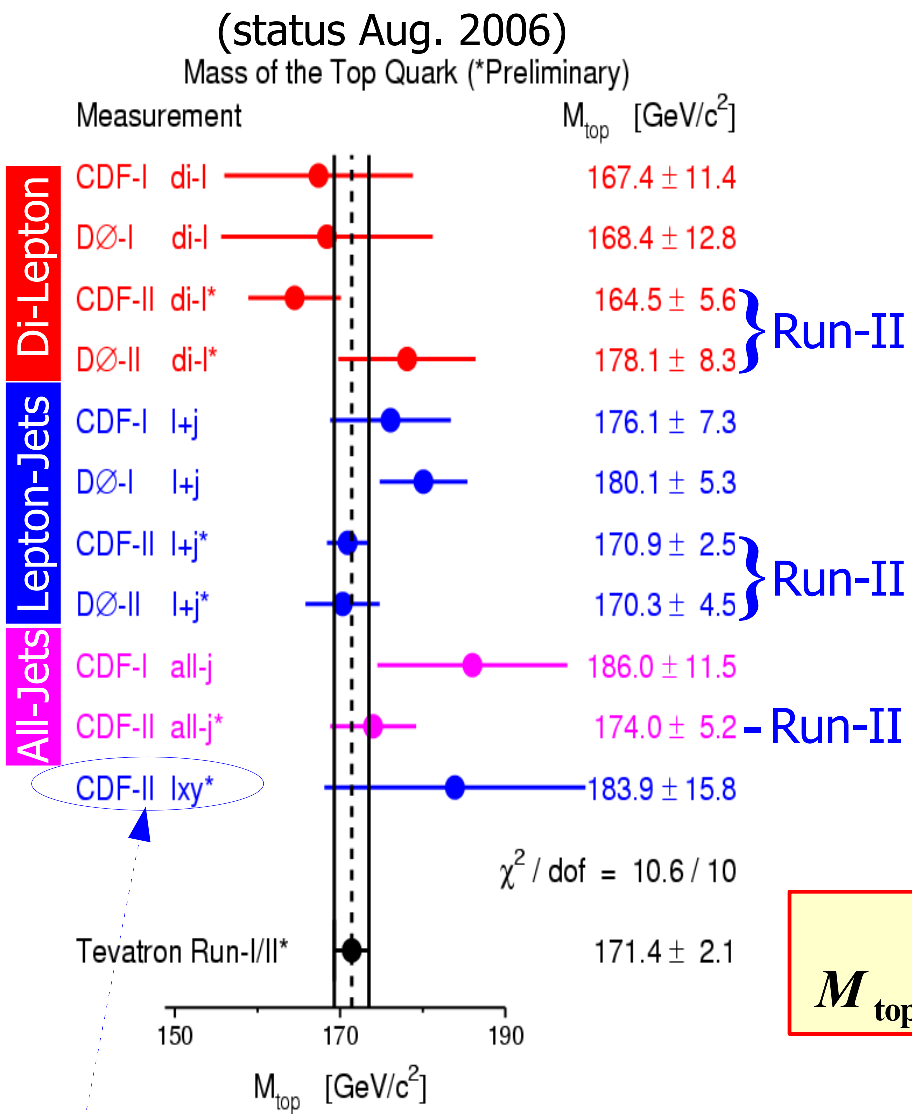
(status Aug. 2006)

- Detailed comparison taking correlations between systematic uncertainties into account

→ Results from different channels are consistent!

- DØ will present new 1/fb results soon.

Tevatron Combination



- Significant improvements w.r.t. Run-I.

Combination of best individual results using BLUE technique: ("Best Linear Unbiased Estimate", NIM A270 110, A500 391)

- Account for correlations
- Include Run-I results

(New CDF All-Jets result not yet incorporated.)

Tevatron combined (status Aug. 2006)

$$M_{top} = 171.4 \pm 1.2\,(\text{stat.}) \pm 1.4\,(\text{JES}) \pm 1.0\,(\text{syst.})\ \text{GeV}/c^2$$

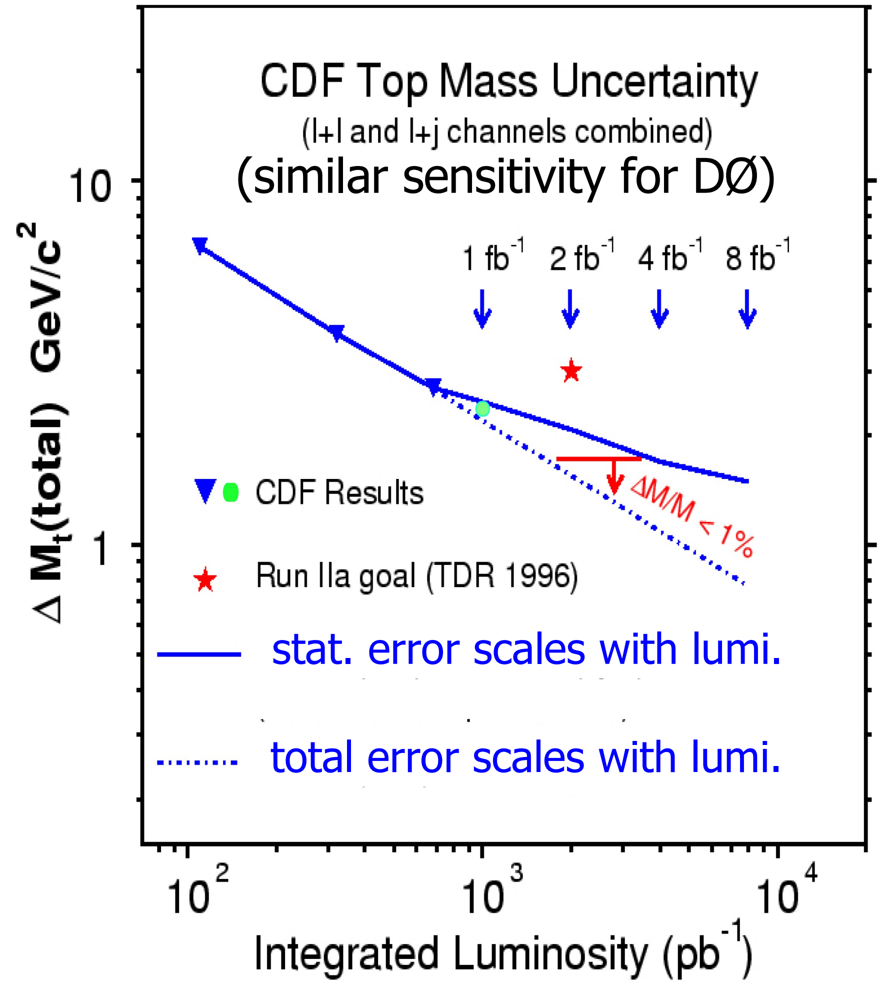
Decay length technique: systematics uncorrelated with other measurements, promising for LHC (see appendix).

Non-JES will be limiting factor at the end of Run-II (see appendix).

Conclusions and Outlook

- Confidence through consistent picture of many excellent top mass determinations.
- Important lesson: JES uncertainty can be greatly reduced by in-situ W calibration.
- CDF&DØ have reached a combined precision of 1.2% (better than Run-IIa goal).

$$M_{top} = 171.4 \pm 2.1\,\mathrm{GeV}/c^2$$



- Can reach 1% precision with full Run-II data, may even push to $\Delta m_{top} \sim 1\mathrm{GeV}/c^2$ (expected after 5-10 years LHC!)

Tevatron might be the lasting legacy for the top quark mass!

(...at least for a while)

Backup Slides

Systematics

(status 03/07/2007)

physics model

Uncertainties [GeV/c²]	Di-Lepton (ME 1030 pb^{-1})	Lepton+Jets (ME 955 pb^{-1})	All-Jets (TM 940 pb^{-1})
Statistical	3.9	1.6	2.8
JES	3.5	1.5	2.4
Residual JES		0.4	0.7
b-JES		0.6	0.4
ISR/FSR	0.4	1.1	1.2
PDF	0.8	0.1	0.5
Generator	0.9	0.2	1.0
MC statistics	0.7	0.2	0.4
Background model	0.2		0.9
Sample composition	0.7		0.1
Lepton p_T	0.1	0.2	
b-tag p_T dep.		0.3	
Multiple interactions	0.2	0.1	
Method	0.6		0.2
Total systematics (excluding JES)	1.7	1.4	2.1

Lepton+Jets (ME 370 pb^{-1})

Source of Uncertainty	*b*-Tagging Analysis
Statistical uncertainty and jet energy scale	+4.1 −4.5
JES only	3.5
Physics modeling:	
Signal modeling	±0.46
Background modeling	±0.40
PDF uncertainty	+0.16 −0.39
b fragmentation	±0.56
b/*c* semileptonic decays	±0.05
Detector modeling:	
$JES\ p_T$ dependence	±0.19
b response (h/e)	+0.63 −1.43
Trigger	+0.08 −0.13
b tagging	±0.24
Method:	
Signal fraction	±0.15
QCD contamination	±0.29
MC calibration	±0.48
Total systematic uncertainty	+1.2 −1.8
Total uncertainty	+4.3 −4.9

- Non-JES systematics mainly dominated by physics model:
  - amount of FSR gluon radiation, hadronization model,...

... will limit or knowledge of M_{top} in future!

CDF: Template, Di-Lepton, 1030pb^{-1}

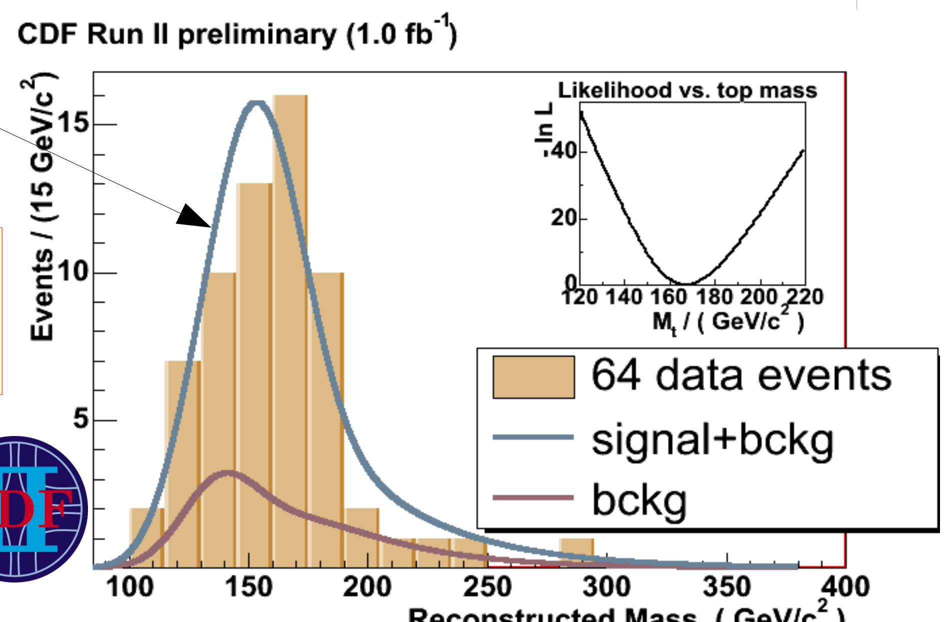
- Under-constrained problem requires assumption for one kinematic variable...
  here: longitudinal momentum P_z of $t\bar{t}$ system

$$
\begin{aligned}
P_{\nu_{1x}} + P_{\nu_{2x}} &= \not{E}_{T_x} \\
P_{\nu_{1y}} + P_{\nu_{2y}} &= \not{E}_{T_y} \\
P_{tz} + P_{\bar{t}z} &= P_{t\bar{t}z} \\
M_t &= M_{\bar{t}} \\
M_W &= 80.4 \\
\vec{P}_b + \vec{P}_{W+} &= \vec{P}_t \\
\vec{P}_{\bar{b}} + \vec{P}_{W-} &= \vec{P}_{\bar{t}} \\
\vec{P}_{l+} + \vec{P}_{\nu} &= \vec{P}_{W+} \\
\vec{P}_{l-} + \vec{P}_{\bar{\nu}} &= \vec{P}_{W-}
\end{aligned}
$$

- Assume $P_z(t\bar{t})=0$, $\sigma\{P_z(t\bar{t})\}=180\text{GeV}/c^2$:
  No top mass dependence, same for signal and background
  ...derived from MC and lepton plus jets data;
- Solve numerically equations within allowed phase space:
  For each event, dice 10K times the two b-quark energies, E_T(miss), and $P_z(t\bar{t})$ around their measured/assumed values within their given resolutions.
- Sum up and take the most probable resulting ("raw reconstructed") top quark mass to build the template.

- No in-situ JES calibration.

MC expected

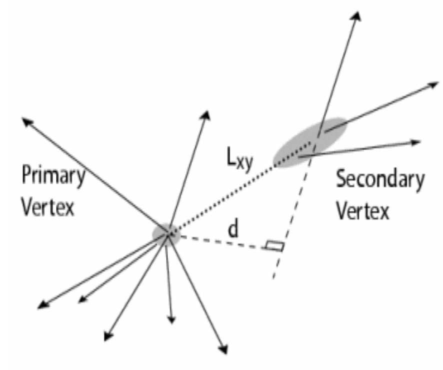


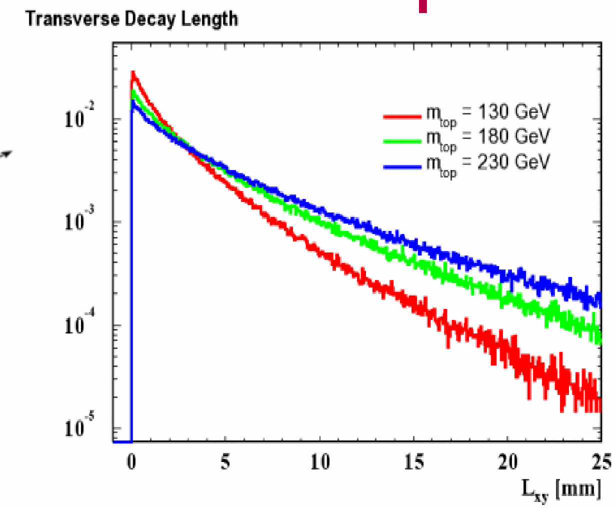
Result using 64 candidate events (≥0 b-tag):

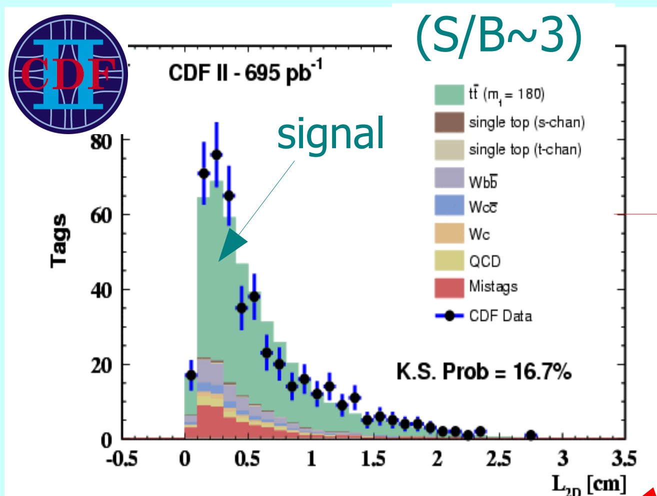
$$M_{top} = 168.1\,^{+5.6}_{-5.5}\,(\text{stat.}) \pm 3.2\,(\text{JES}) \pm 2.4\,(\text{syst.})\,\text{GeV}/c^2$$

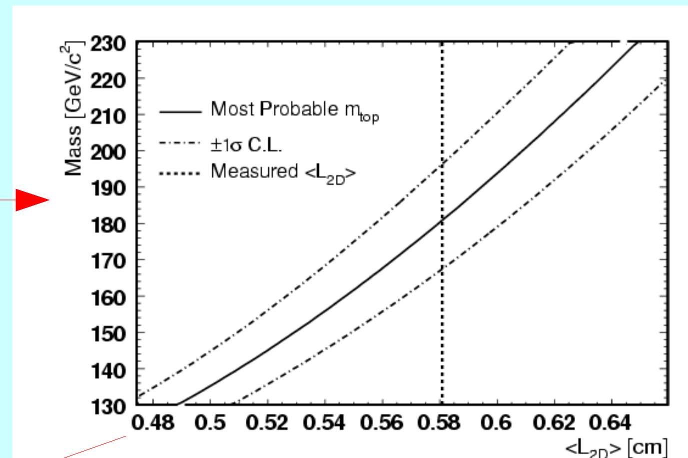
CDF: Decay Length Technique, Lepton+Jets 695pb^{-1}

- Lepton+jets (with ≥ 3 jets, ≥ 1 b-tag)
- Template variable is transverse decay length
- Top mass sensitivity comes through slope of an exponential curve (difficult to measure)
- Mean of decay length is converted to most probable top mass (assessed via MC)



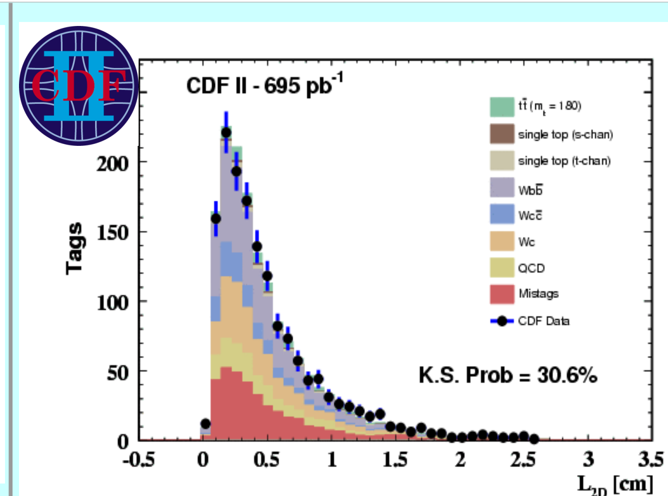






Result using 375 candidate events:

$$M_{top} = 183.9^{+15.7}_{-13.4}(\text{stat.}) \pm 0.3(\text{JES}) \pm 5.6(\text{syst.})\,\text{GeV}/c^2$$



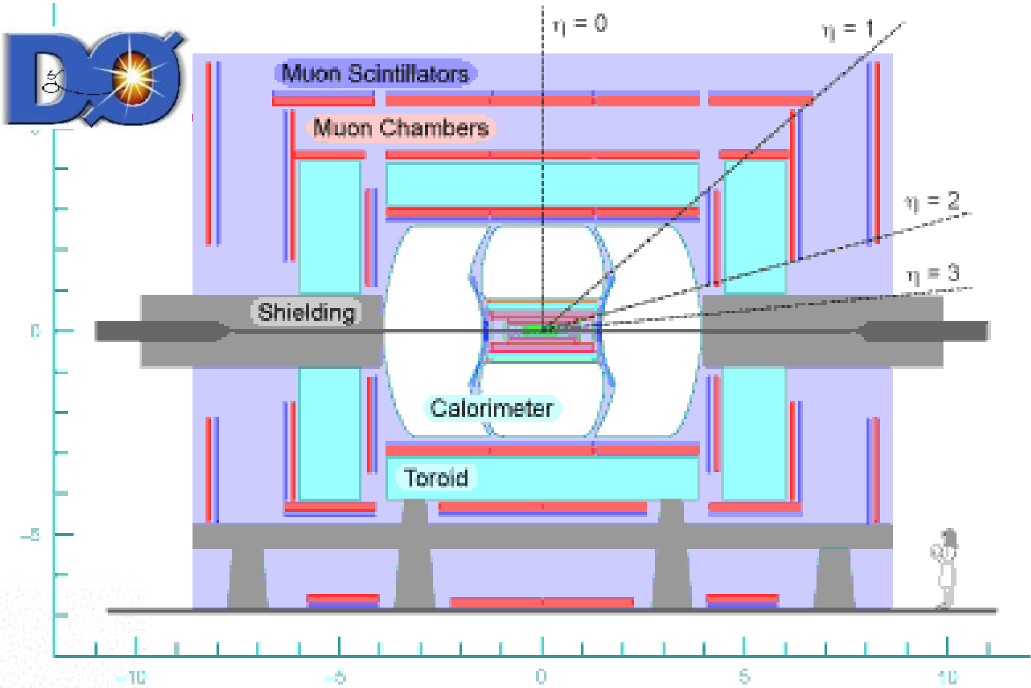
control sample (W+1jet,W+2jets)

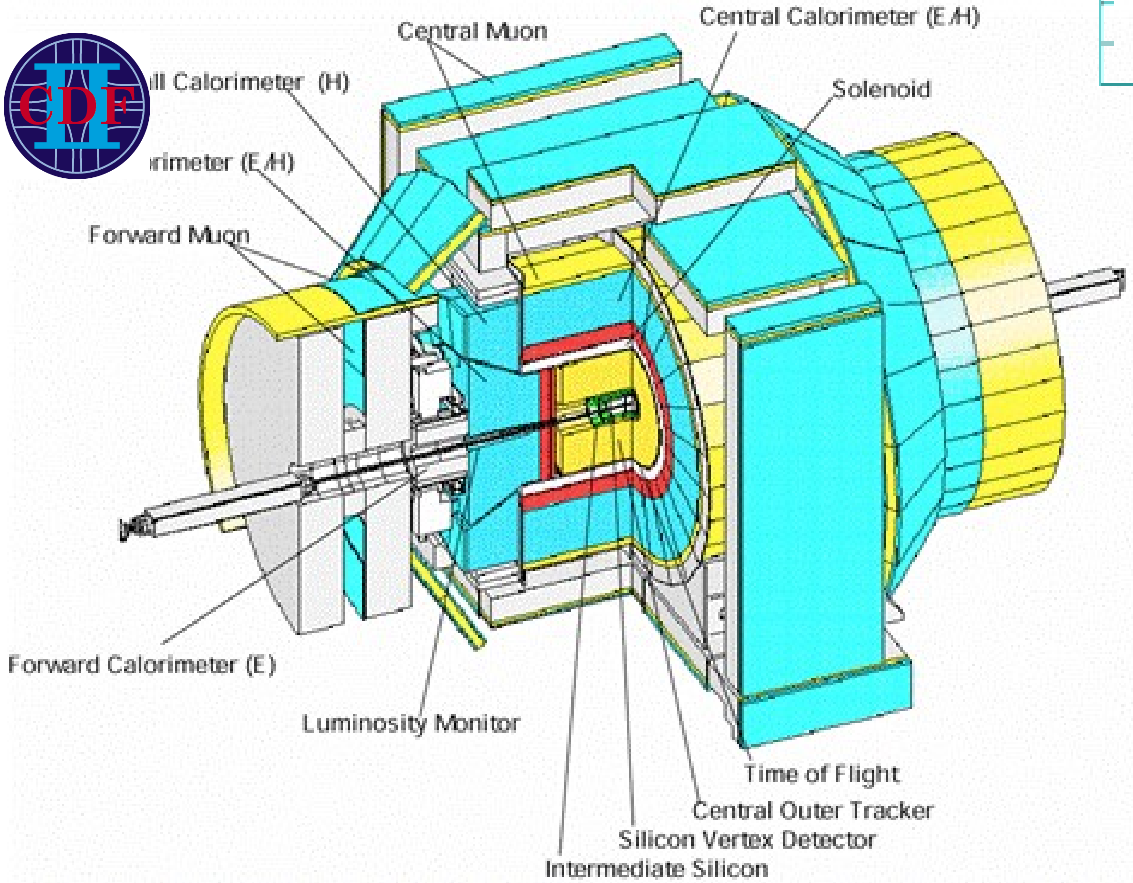
- Systematics largely uncorrelated with those of other measurements!
- Statistics limited, but can make significant contribution to LHC

CDF and DØ Detectors

Hermetic multi-purpose detectors:

- **Precision tracking**
- **Calorimetry (EM, HAD)**
- **Muon system**
- **Vertex detectors**





SecVtx b-quark tagging (CDF example):

- $L_{xy}/\sigma > 3$ ($\sigma \sim 150\mu m$)
- $t\bar{t}$ tag efficiency ~ 55%
- $t\bar{t}$ fake rate ~ 0.5 %