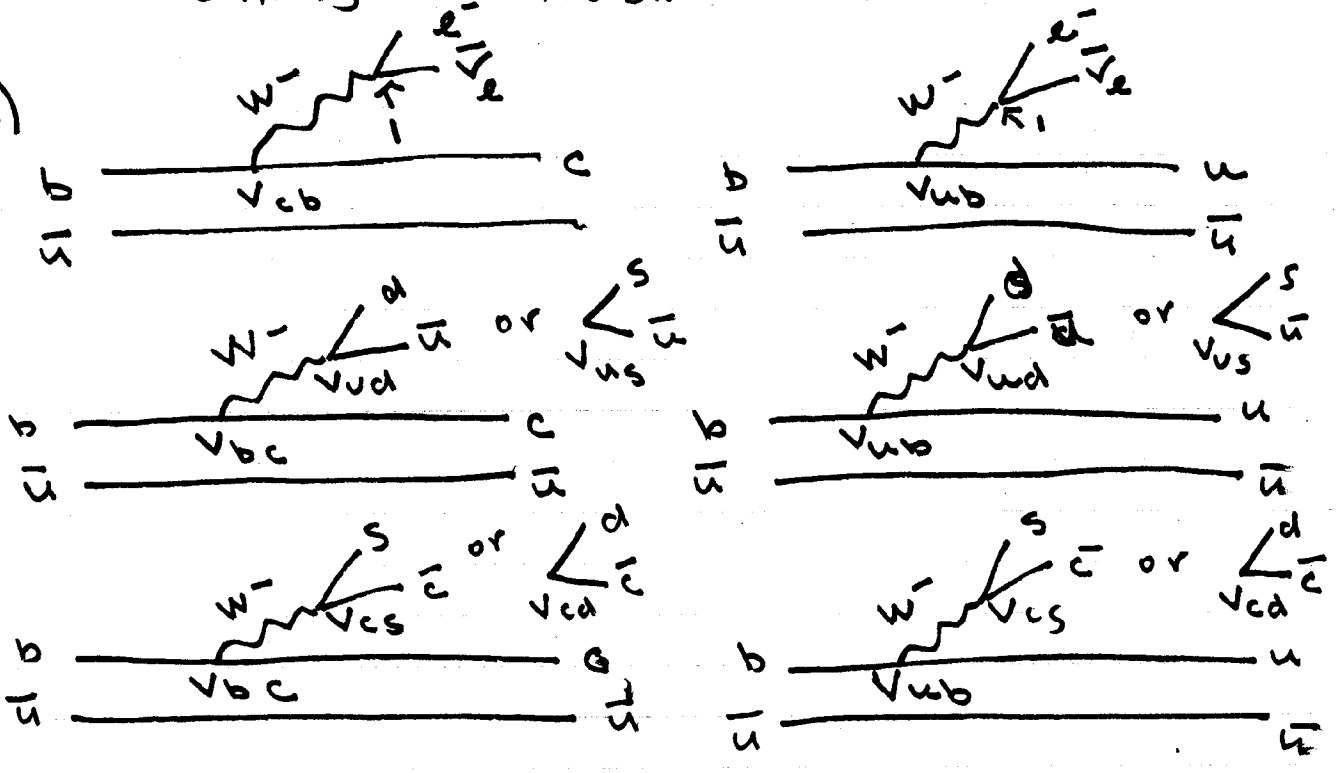
Physics 129

Solutions to Problem Set #9

1 a)



b) $BR(B^- \to X e \nu_e) = \dfrac{1^{st} \text{ row above with } \ell = e}{1^{st} \text{ row} * 3 \text{ flavors} + 2^{nd} \text{ row} * 3 \text{ colors} + 3^{rd} \text{ row} * 3 \text{ colors}}$

Where I have ignored the difference in phase space for the diagrams.

Note:

$$|V_{ud}|^2 + |V_{us}|^2 \sim 1 \quad \text{and}$$

$$|V_{cs}|^2 + |V_{cd}|^2 \sim 1$$

$$\therefore \quad BR \cong \frac{1}{3+3+3} \sim \frac{1}{9}$$

c) $\Gamma(B \to X e \bar{\nu}) = (|V_{bc}|^2 + |V_{bu}|^2) \dfrac{m_b^5}{m_\mu^5} \dfrac{1}{\tau_\mu}$

also $\Gamma(B \to X e \bar{\nu}) = \Gamma_{TOT} \; BR(B \to X e \nu)$

$= \dfrac{BR(B \to X e \nu)}{\tau_{B^-}}$

$\therefore (|V_{bc}|^2 + |V_{bu}|^2) =$

$$\frac{1/9}{1.65 \times 10^{-12}\,s} \left(\frac{105}{5000}\right)^5 2.2 \times 10^{-6}\,s$$

$= 6 \times 10^{-4}$

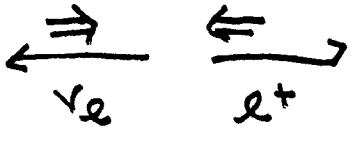
Unitarity of CKM matrix $\Rightarrow$

$|V_{bc}|^2 + |V_{bu}|^2 + |V_{tb}|^2 = 1$

$\Rightarrow |V_{tb}|^2 = 1 - 6 \times 10^{-4}$

$|V_{tb}| = .9997$

2a) B has spin 0 so



ν_e must be left handed

Conservation of ∡ mom means e^+ must be left handed

But W wants to couple to right handed antiparticle $\Rightarrow$ "helicity suppressed"

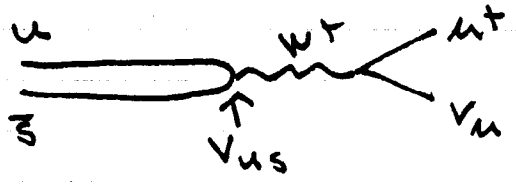
b) From Perkins pg 212

$$BR(B^+ \to \ell^+ \nu_\ell) \sim m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

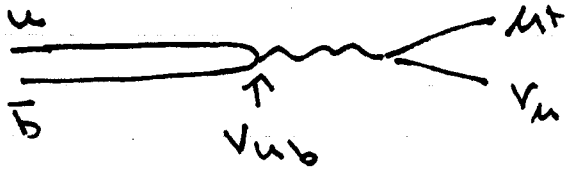
$\therefore$ $BR(\tau^+ \nu_\tau) : BR(\mu^+ \nu_\mu) : BR(e^+ \nu_e)$

$= 28 : 0.011 : 2.5 \times 10^{-7}$

c) $K^+ \to \mu \nu_\mu$



$B^+ \to \mu \nu_\mu$



$\Gamma(\text{meson}^+ \to \mu^+ \nu_\mu) \sim$ phase space $*$ $|\text{CKM factor}|^2 * |\psi(0)|^2$

if we assume $|\psi(0)|^2 / M^2 \sim$ const (as was true for Perkins page 125)

$$\Gamma \sim m_\ell^2 (1 - m_\ell^2 / m^2) |V_{ij}|^2 m^2$$

$$\frac{\Gamma(B^+ \to \mu\nu)}{\Gamma(K^+ \to \mu\nu)} = \frac{(1 - m_\mu^2 / m_B^2)^2}{(1 - m_\mu^2 / m_K^2)^2} \left|\frac{V_{ub}}{V_{us}}\right|^2 \frac{m_B^2}{m_K^2}$$

$$\Gamma(B^+ \to \mu^+ \nu) = .0476\, \Gamma(K^+ \to \mu^+ \nu)$$

$$= (.0476) \frac{(.635)}{1.24 \times 10^{-8}\, s} = 2.4 \times 10^6\, s^{-1}$$

d) But $\Gamma_B^{TOT} = \frac{1}{\tau_B} = \frac{1}{1.65 \times 10^{-12}\,s} =$

$\therefore \frac{\Gamma(B^+ \to \mu\nu)}{\Gamma(B^+ \to \text{anything})} = (1.65 \times 10^{-12}\,s)(2.4 \times 10^{6}\,s^{-1})$

$= 4 \times 10^{-6}$

e) $\mathcal{L} = 3 \times 10^{33}\ cm^{-2}\,s^{-1}$

$\sigma = 1\,nb = 10^{-37}\ m^2 = 10^{-33}\ cm^2$

$\# = \mathcal{L}\sigma t = (3 \times 10^{33}\ cm^{-2}\,s^{-1})(10^{-33}\ cm^2)(10^7\,s/yr)$

$= 3 \times 10^7$ per year $(B^+B^- + B^0\bar{B}^0)$

$\#(B^+ \to \mu\nu) = (4 \times 10^{-6})(3 \times 10^7)/2$

$= \frac{120}{2}/yr = 60/yr$

$\Rightarrow$ 2 months

3. Perkins 7.2

a The Lorentz transformation tells us:

$E_\nu^{rest} = \gamma(E_\nu^{lab} - \beta p_x^{lab}) = \gamma(E_\nu^{lab} - \beta E_\nu^{lab}\cos\theta)$

$E_\nu^{rest} = \gamma E_\nu^{lab}(1 - \beta\cos\theta)$

$E_\nu^{rest} = \frac{E_\nu^{lab}}{\gamma}(\gamma^2 - \beta\gamma^2\cos\theta)$

now, Taylor expand $\cos\theta = 1 - \theta^2/2 + \ldots$

$E_\nu^{rest} = \frac{E_\nu^{lab}}{\gamma}(\gamma^2 - \beta\gamma^2(1 - \theta^2/2)) = \frac{E_\nu^{lab}}{\gamma}(\gamma^2(1-\beta) + \beta\gamma^2\frac{\theta^2}{2})$

3a/cont

$$E_\nu^{rest} = \frac{E_\nu^{lab}}{\gamma}\left(\frac{1-\beta}{1-\beta^2} + \beta\gamma^2\frac{\theta}{2}\right) = \frac{E_\nu^{lab}}{\gamma}\left(\frac{1}{1+\beta} + \beta\gamma^2\frac{\theta}{2}\right)$$

for $\beta \approx 1$

$$E_\nu^{rest} = \frac{E_\nu^{lab}}{\gamma}\left(\frac{1}{2} + \frac{\beta\gamma^2\theta}{2}\right) = \frac{E_\nu^{lab}}{2\gamma}\left(1 + \gamma^2\theta\right)$$

$$\therefore E_\nu^{lab} = \frac{2\gamma E_\nu^{rest}}{1+\gamma^2\theta^2}$$

But E_ν^{rest} given by $E_\pi = E_\nu + E_\mu$

$$m_\pi = E_\nu + \sqrt{m_\mu^2 + p_\mu^2} = E_\nu + \sqrt{m_\mu^2 + E_\nu^2}$$

where the last step uses $\vec{p}_\mu = -\vec{p}_\nu$ and $|p_\nu| = E_\nu$

$$\therefore m_\pi - E_\nu = \sqrt{m_\mu^2 + E_\nu^2}$$

$$m_\pi^2 - 2m_\pi E_\nu + E_\nu^2 = m_\mu^2 + E_\nu^2$$

$$m_\pi^2 - m_\mu^2 = 2m_\pi E_\nu \Rightarrow E_\nu^{rest} = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

$$E_\nu^{lab} = \frac{2\gamma}{1+\gamma^2\theta^2}\left(\frac{m_\pi^2 - m_\mu^2}{2m_\pi}\right) = \frac{\gamma(m_\pi^2 - m_\mu^2)}{m_\pi(1+\gamma^2\theta^2)}$$

for kaon, substitute m_K for m_π

b) $E_\nu^{lab}(max)$ is when $\theta = 0$ so $= 2\gamma E_\nu^{rest}$

$$E_\nu^{lab}(max) = \frac{\gamma(m_\pi^2 - m_\mu^2)}{m_\pi}$$

$E_\nu^{lab}(min)$ corresponds to case where ν is going in $-z$ direction in center-of-mass.

$$E_\nu^{lab} = \gamma(E_\nu^{rest} - \beta E_\nu^{rest}) = \gamma(1-\beta)E_\nu^{rest}$$

for $\beta \sim 1$ $\quad E_\nu^{lab}(min) \sim 0$

3 c) The radius of detector = 2 m and distance to detector is 400 m so $\tan\theta \sim \sin\theta \sim \theta = 2/400$
for Kaon with $p = 200$ GeV

$$\beta\gamma m_K = 200 \text{ GeV} \Rightarrow \beta\gamma = \frac{200 \text{ GeV}}{.493 \text{ GeV}} = 405$$

Since $\beta \sim 1$, $\gamma = 405$.

$$\therefore E_\nu(\text{max}) \text{ from Kaons} = 405\left(\frac{(.493)^2 - (.105)^2}{.493}\right)$$

$E_\nu(\text{max})$ from Kaons = 191 GeV

~~from ... = 84~~

for $\theta = 2/400$ $\quad E_\nu = \dfrac{191}{1 + (405)^2 (2/400)^2} = 37$ GeV

For Pions of 200 GeV $\quad \beta\gamma = \dfrac{200 \text{ GeV}}{.140} = 1428$

$$\therefore E_\nu(\text{max}) = 1428\left(\frac{(.140)^2 - (.105)^2}{.140}\right) = 84 \text{ GeV}$$

for $\theta = 2/400$ $\quad E_\nu = \dfrac{84}{1 + (1428)^2 (2/400)^2} = 1.6$ GeV

d) fraction of π decaying $= 1 - e^{-t/\tau_{lab}} = 1 - e^{-t/\gamma\tau_{rest}}$

$= 1 - e^{-\ell/\gamma c\tau_{rest}} = 1 - e^{-0.0242}$

$= 0.0239$

decaying $= 10^{10} \times .0239 = 2.39 \times 10^8$

e) ν spectrum for π goes from 0-84 GeV
average is 42 GeV.

$\therefore \sigma = .6(42) \times 10^{-38} \text{ cm}^2 = 2.5 \times 10^{-37} \text{ cm}^2$

density of steel $= 8 \text{ g/cm}^3$

Since there are N_0 nucleons per gram of material ($N_0 = 6 \times 10^{23}$) if the length of our target is ℓ there are $N_0 \dfrac{\text{nucleons}}{\text{gm}} \times 8 \dfrac{\text{gm}}{\text{cm}^3} \times \ell \dfrac{\text{nucleons}}{\text{cm}^2}$

here ℓ can be obtained using

$$\pi \underbrace{(2\text{m})^2}_{\times 100 \frac{\text{cm}}{\text{m}}} \ell \times 8 \frac{\text{g}}{\text{cm}^3} = 100 \text{ ton} = 10^8 \text{ gm}$$

3e) cont

$\therefore \; R = 10^4 / 32\pi \text{ cm} \sim 100 \text{ cm}$

$$\frac{\#\ \text{interactions}}{\nu} = 6\times10^{23}\ \frac{\text{nucl}}{\text{gm}} \times 8\ \frac{\text{gm}}{\text{cm}^3} \times 100\text{ cm} \times 2.5\times1$$

$$= 1.2 \times 10^{-10}\ \text{int}/\nu$$

So # interactions $= 1.2\times10^{-10} \times 2.4\times10^{8}\ \nu/\text{burst}$

$= 2.8\times10^{-2}$ int / burst

f) The 350 m of absorber is needed to stop the μ before the detector

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