Midterm 1 Sample Exam Solutions

Physics 7B (Section 1) Spring 2002
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Name :

Student ID Number :

Discussion Section :

Name of Teaching Assistant :

DO NOT OPEN THIS EXAM
UNTIL YOU ARE TOLD TO DO SO !!

Problem	Points	Score
1	8	
2	34	
3	12	
4	12	
5	34	
Total	100	

Some equations and numbers that might be useful.

- Ideal Gas Law :

$$PV = nRT$$

  - Universal gas constant, R

$$\begin{aligned} R &= 8.315\ J/(mol \cdot K) \\ &= 0.0821\ (L \cdot atm)/(mol \cdot K) \\ &= 1.99\ calories/(mol \cdot K) \end{aligned}$$

  - the number of moles, $n = N/N_A$
  - the number of molecules, N
  - Avogadro's number, $N_A = 6.02 \times 10^{23}$
  - Boltzmann's constant : $k = R/N_A$

- Internal Energy of an Ideal Gas :

$$U = \frac{3}{2}NkT = \frac{3}{2}nRT$$

- Specific Heat, c = amount of heat required to raise the temperature of unit mass by 1 C^o.

$$Q = mc\Delta T$$

  Q = Amount of Heat required to change the temperature of a given material with the mass m by ΔT.

- Molar Specific Heats for Gases, C_V and C_P, heat required to raise 1 mol of the gas by 1 C^o at constant volume and at constant pressure, repectively.

$$\begin{aligned} Q &= nC_V\Delta T \text{ (volume constant)} \\ Q &= nC_P\Delta T \text{ (pressure constant)} \\ C_P &= C_V + R \\ \gamma &= \frac{C_P}{C_V} \end{aligned}$$

- Adiabatic Expansion of a Gas

$$PV^\gamma = \text{constant} \tag{0.1}$$

- Heat Transfer: Conduction, Convection, Radiation

$$\begin{aligned} \frac{\Delta Q}{\Delta t} &= kA\frac{T_1 - T_2}{l} \text{ (conduction)} \\ \frac{\Delta Q}{\Delta t} &= e\sigma A(T_1^4 - T_2^4) \text{ (radiation)} \end{aligned}$$

- Entropy Change of Irreversible Heat Transfer
  e.g. an Entropy change ΔS of an object with mass m and specific heat c whose intitial temperature is T_i and whose final temperature is T_f :

$$\Delta S = mc \ln \frac{T_f}{T_i}$$

Problem #1 (8 points)

A steel metric scale is to be ruled so that the millimeter intervals are accurate to within 5×10^{-5} mm at a certain temperature. What is the maximum temperature variation allowable during the ruling ?

Solution :
The themal conductivity of steel = 11×10^{-6} C^o.

$$\begin{aligned}
\Delta L &= \alpha L \Delta T \\
\Delta T &= \frac{\Delta L}{\alpha L} \\
&= \frac{5 \times 10^{-5}\ mm}{(11 \times 10^{-6}) \cdot (1.0\ mm)} \\
&= 4.5\ C^o
\end{aligned}$$

The temperature during the ruling must be kept constant to within about 5 C^o, and the scale must be used within that same interval of temperature at which it was made.

Problem #2 (34 points)

A sample of gas consisting of 0.11 mol is compressed from a volume of $V_i = 4.0$ m^3 to $V_f = 1.0$ m^3 along two paths (path 1 and path 2) while its pressure increases from 10 to 40 Pa. Path 1 represents an isothermal process. Path 2 consists of two processes, one at constant pressure followed by another at constant volume. Assume that the gas is ideal.

1. (8 points) Calculate the work done by the gas along the two different paths.

   (a) path 1 :
   T is constant, thus

   $$P_i V_i = P_f V_f = nRT. \tag{0.2}$$

   The work done during the isothermal process is

   $$\begin{aligned} W_1 &= nRT\ ln\frac{V_f}{V_i} \\ &= P_i V_i\ ln\frac{V_f}{V_i} \\ &= (10\ Pa)(4.0\ m^3)\ ln\frac{1.0\ m^3}{4.0\ m^3} \\ &= -55\ J \end{aligned}$$

   Negative work.

   (b) path 2 :

   i. Work done at constant pressure (isobaric process) :

   $$\begin{aligned} W &= P_i\ (V_f - V_i) \\ &= (10\ Pa)\ (1.0\ m^3 - 4.0\ m^3) \\ &= -30\ J \end{aligned}$$

   ii. Work done at constant volume (isochoric process) is zero.

   $$W = 0$$

   The work done by the gas along path 2 is

   $$W_2 = -30\ J$$

   and negative.

2. (6 points) What is the heat added to or removed from the gas in the two paths ? You should clarify the direction of heat.

   (a) path 1 :

   Since temperature is the same along path 1, there is no change in internal energy. Thus,

$$\Delta U = 0 = W_1 - Q_1$$

   and the work done by the gas is the heat added to the gas.

$$Q_1 = W_1 = -55\ J$$

   Since Q_1 is negative, the heat of 55 J is removed from the gas in path 1.

   (b) path 2 :

   The initial and final points in path 2 are at the same temperature, thus there is no change in internal energy;

$$Q_2 = W_2 = -33\ J$$

   Since Q_2 is negative, the heat of 33 J is removed from the gas in path 2.

3. (8 points) Let the cylinder be removed from the thermal reservior and let us compress the gas reversely and adiabatically until its volume is $V_f = 1.0$ m^3. Calculate the pressure at $V_f = 1.0$ m^3 ? Assume the gas to be Helium (a monatomic gas with $\gamma = 1.66$).

Solution :

P_f can be found from the adiabatic relationship

$$\begin{aligned} P_i V_i^{\gamma} &= P_f V_f^{\gamma} \\ P_f &= P_i \cdot (\frac{V_i}{V_f})^{\gamma} \\ &= (10\ Pa) \cdot (\frac{4.0\ m^3}{1.0\ m^3})^{1.66} \\ &= 100\ Pa \end{aligned}$$

4. (4 points) Sketch the P-V diagram of part 3.

5. (8 points) What is the change in internal energy of the gas in part 3 ?

<u>Solution</u>

Since the gas is monatomic, the change in internal energy can be written as

$$\begin{aligned} \Delta U &= \frac{3}{2} nR\Delta T \\ &= \frac{3}{2} nR(T_f - T_i) \end{aligned}$$

Using the ideal gas law, we can find T_i,

$$\begin{aligned} P_i V_i &= nRT_i \\ T_i &= (P_i V_i)/(nR) \end{aligned}$$

and the final temperature T_f,

$$T_f = (P_f V_f)/(nR).$$

The change in internal energy is then

$$\begin{aligned} \Delta U &= \frac{3}{2} nR(T_f - T_i) \\ &= \frac{3}{2} nR(\frac{P_f V_f}{nR} - \frac{P_i V_i}{nR}) \\ &= \frac{3}{2}(P_f V_f - P_i V_i) \\ &= \frac{3}{2}(100\ Pa \cdot 1\ m^3 - 10\ Pa \cdot 4\ m^3) \\ &= 90\ J \end{aligned}$$

Problem #3 (12 points)

You wish to increase the efficiency of an ideal gas engine. Is it better to raise the temperature of the high-temperature reservoir by a certain amount, or lower the temperature of the low-temperature reservoir by the same amount ? Explain your logic.

Solution :
The efficiency of an ideal gas engine is

$$\begin{aligned} \epsilon &= \frac{W}{Q_H} \\ &= \frac{Q_H - Q_L}{Q_H} \\ &= 1 - \frac{Q_L}{Q_H} \\ &= 1 - \frac{T_L}{T_H} \end{aligned}$$

The efficiency goes up if we lower T_L or we raise T_H.

1. Let's lower T_L by ΔT;

$$\epsilon_1 = 1 - \frac{T_L - \Delta T}{T_H}$$

2. Let's raise T_H by ΔT;

$$\epsilon_2 = 1 - \frac{T_L}{T_H + \Delta T}$$

Let's now compare ϵ_1 to ϵ_2.

$$\begin{aligned} \epsilon_1 - \epsilon_2 &= 1 - \frac{T_L - \Delta T}{T_H} - (1 - \frac{T_L}{T_H + \Delta T}) \\ &= \frac{-T_L + \Delta T}{T_H} + \frac{T_L}{T_H + \Delta T} \\ &= \frac{(-T_L + \Delta T)(T_H + \Delta T) + T_L T_H}{T_H(T_H + \Delta T)} \\ &= \frac{\Delta T(T_H - T_L) + (\Delta T)^2}{T_H(T_H + \Delta T)} \end{aligned}$$

Since $T_H > T_L$ and $(\Delta T)^2 > 0$, $\epsilon_1 > \epsilon_2$.
Therefore, it is better to lower the temperature of the low-temperature reservoir.

Problem #4 (12 points)

A Carnot engine works between temperatures T_1 and T_2 with $T_1 > T_2$. It drives a Carnot refrigerator that works between two different temperatures T_3 and T_4 with $T_3 > T_4$. Find the ratio $|Q_3|/|Q_1|$ in terms of the four temperatures.

Solution

First, consider a Carnot engine between T_1 and T_2 with $T_1 > T_2$. The work done by this engine is

$$W = |Q_1| - |Q_2| \tag{0.3}$$

where the heat flows from the T_1 reservoir to the T_2 reservoir. Since this is an ideal engine, we can get a relationship

$$|Q_2|/|Q_1| = T_2/T_1$$

The exact amount of the work done by this engine drives the refrigerator that works between T_3 and T_4 with $T_3 > T_4$. In the refrigerator, the heat flows from the lower-temperature reservoir to the higher-temperature reservoir.

$$\begin{aligned} |Q_3| &= |Q_4| + W \\ W &= |Q_3| - |Q_4| \end{aligned} \tag{0.4}$$

From Eq.s 0.3 and 0.4,

$$\begin{aligned} |Q_3| - |Q_4| &= |Q_1| - |Q_2| \\ |Q_3|(1 - |Q_4|/|Q_3|) &= |Q_1|(1 - |Q_2|/|Q_1|) \\ |Q_3|(1 - T_4/T_3) &= |Q_1|(1 - T_2/T_1) \\ |Q_3|/|Q_1| &= (1 - T_2/T_1)/(1 - T_4/T_3) \end{aligned}$$

Problem #5 (34 points)

A hot iron horseshoe with mass of 0.40 kg which has just been forged, is dropped into 1.35 L of water in a 0.30-kg iron pot initially at 20^{o}C. Ignore any heat transfer between the environment (air) and these objects.

1. (10 points) If the final equilibrium temperature is 25^{o}C, what is the initial temperature of the hot horseshoe ?

   Solution

   Since all the energy that leaves one object in this isolated system enters other objects in the system, conservation of heat energy requires

   $$Q_{water} + Q_{pot} + Q_{horseshoe} = 0$$

   $$m_w c_w (T_e - 20^oC) + m_{pot} c_{iron} (T_e - 20^oC) + m_{horseshoe} c_{iron} (T_e - T_{horseshoe}) = 0$$

   where

   - m_w = 1 g/cm^3 × (1.35 × 10^3 cm^3) = 1.35 kg,
   - m_{pot} = 0.30 kg,
   - $m_{horseshoe}$ = 0.40 kg,
   - c_w = 4190 J/(kg·K),
   - c_{iron} =
   - T_e = 25^{o}C.

   Then

   $$T_{horseshoe} =$$

2. (20 points) Calculate the entropy change of the entire system.

$$\begin{aligned}
\Delta S &= \Delta S_{system} + \Delta S_{environment} \\
\Delta S_{environment} &= 0 \\
\Delta S &= \Delta S_{system} \\
&= \Delta S_{water+pot+horseshoe} \\
&= \Delta S_{water} + \Delta S_{pot} + \Delta S_{horseshoe} \\
\Delta S_{water} &= m_w c_w \; ln \frac{(273+25)^o K}{(273+20)^o K} \\
\Delta S_{pot} &= m_{pot} c_{iron} \; ln \frac{(273+25)^o K}{(273+20)^o K} \\
\Delta S_{houseshoe} &= m_{horseshoe} c_{iron} \; ln \frac{(273+25)^o K}{(273+T_{horseshoe})^o K}
\end{aligned}$$

You should get a positive entropy change ($\Delta S > 0$).

3. (4 points) Does this process violate the 2nd law of thermodynamics ? Explain your reasons in a couple of sentenses.

   Solution

   The 2nd law of thermodynamics states that the total entropy remains unchanged or increases: it remains unchanged for a reversible process, and increases for an irreversible process.

   The process we considered in this problem is irreversible since it is not done in infinitesimal steps, and we can not restore the system to its original state by reversing the process. Thus we expect an entropy increase, that is, $\Delta S > 0$. Since we've got $\Delta S > 0$, the process did not violate the 2nd law of thermodynamics.