# $C$-Parameter and Jet Broadening at PETRA Energies

O. Biebel(1), P.A. Movilla Fernández(1), S. Bethke(1)
and the JADE Collaboration(2)

**Abstract**

$e^+e^-$ annihilation data recorded by the JADE detector at PETRA were used to measure the $C$-parameter for the first time at $\sqrt{s} = 35$ and 44 GeV. The distributions were compared to a resummed QCD calculation. In addition, we applied extended resummed calculations to the total and wide jet broadening variables, $B_T$ and $B_W$. We combined the results on $\alpha_s$ with those of our previous study of differential 2-jet rate, thrust, and heavy jet mass, obtaining $\alpha_s(35\text{ GeV}) = 0.1448^{+0.0117}_{-0.0070}$ and $\alpha_s(44\text{ GeV}) = 0.1392^{+0.0105}_{-0.0074}$. Moreover power corrections to the mean values of the observables mentioned above were investigated considering the Milan factor and the improved prediction for the jet broadening observables. Our study, which considered $e^+e^-$ data of five event shape observables between $\sqrt{s} = 14$ and 183 GeV, yielded $\alpha_s(M_{Z^0}) = 0.1177^{+0.0035}_{-0.0034}$.

(1) III. Physikalisches Institut der RWTH Aachen, D-52056 Aachen, Germany
contact e-mail: Otmar.Biebel@Physik.RWTH-Aachen.DE
(2) for a full list of members of the JADE Collaboration see Reference [1]

# 1 Introduction

In a recent publication [2] we have presented a study on event shapes observables and determinations of $\alpha_s$ using data of $e^+e^-$ annihilations at $\sqrt{s} = 22$ to 44 GeV recorded with the former JADE detector [3] at the PETRA collider. This study provided valuable information which was not available before from $e^+e^-$-annihilations in the PETRA energy range. The results on $\alpha_s$, obtained in a manner similar to the determinations at LEP, demonstrated that the energy dependence of $\alpha_s(Q)$ is in good agreement with the prediction of Quantum Chromodynamics (QCD). Evolved to the $Z^0$ mass scale, the results agree well with those obtained at LEP, and are of similar precision. In addition, power corrections, applied to analytic QCD calculations of the mean values of event shape distributions, were found to qualitatively and quantitatively describe the effects of hadronisation. Thus QCD could be tested without the need of phenomenological hadronisation models.

Meanwhile the perturbative calculations for the jet broadening variables were improved by including a proper treatment of the quark recoil [4]. Furthermore, a resummation of leading and next-to-leading logarithm terms to all orders of $\alpha_s$ (NLLA) became available for the $C$-parameter [5]. Besides these advances in the perturbative description of event shape observables progress was made in the understanding of non-perturbative power corrections to the event shape observables and their mean values. In particular two-loop calculations of such corrections were performed which modify the one-loop result of the power correction to the event shapes by a factor (Milan factor) [6]. Revisiting the interdependence between perturbative and non-perturbative contributions for the case of jet broadening observables yielded in Reference [7] a further modification of the results given in [4].

In this paper we complement our previous publication [2] by a new $\mathcal{O}(\alpha_s^2)$+NLLA determination of $\alpha_s$ from $C$-parameter and update our $\alpha_s$ determination from jet broadening at $\sqrt{s} = 35$ and 44 GeV in the Sections 3 and 4. We also applied power corrections to the $C$-parameter distributions and re-investigated those for the mean values of the thrust, heavy jet mass and both jet broadening observables in Section 5. We start in Section 2 with a brief summary of the data samples used and draw conclusions from our results in Section 6.

# 2 JADE data

The JADE detector was operated from 1979 until 1986 at the PETRA electron-positron collider at centre-of-mass energies of $\sqrt{s} = 12$ to 46.7 GeV. A detailed description of the JADE detector can be found in [1, 3]. The main components of the detector were the central jet chamber to measure charged particle tracks and the lead glass calorimeter to measure energy depositions of electromagnetic showers, which both covered almost the whole solid angle of $4\pi$.

For the present study we consider data collected by the JADE detector in 1984 to 1986 at centre-of-mass energies between 39.5-46.7 GeV and at 35 GeV. Multihadronic events were selected by the standard JADE selection cuts [8] which have been summarised in detail in our previous publication [2]. Background from two-photon processes and $\tau$-pair events and from events with hard initial state photon radiation were mainly suppressed by posing constraints on the visible energy, the total momentum and the charged particle multiplicity of the event, and by cuts in the polar angle of the thrust axis. The estimated purity of the resulting data samples is about 98% [9]. The numbers of events retained after these cuts are 6158 around $\sqrt{s} = 44$ GeV and 20 926 at $\sqrt{s} = 35$ GeV.

Corresponding *original* Monte Carlo detector simulation data for 35 and 44 GeV were based on the QCD parton shower plus string fragmentation implemented by the JETSET 6.3 event generator [10]. Both samples included a detailed simulation of the acceptance and resolution of the JADE detector. We used the standard set of parameters for the event generation as described in [2]. As pointed out there, the agreement of the simulated data with the measurements is good.

# 3 Measurement of event shapes

From the data passing the selection criteria as described in the previous section, the distributions of the $C$-parameter and the jet broadening event shape variables $B_T$ and $B_W$ were determined using the 3-momenta $\vec{p}_i$ of the reconstructed particles. The $C$-parameter is defined in Reference [11] as

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$$

where $\lambda_\gamma$, $\gamma = 1, 2, 3$, are the eigenvalues of the momentum tensor

$$\Theta^{\alpha\beta} = \frac{\sum_i \vec{p}_i^{\,\alpha} \vec{p}_i^{\,\beta} / |\vec{p}_i|}{\sum_j |\vec{p}_j|} \quad .$$

The definition of the total and wide jet broadening measures can be found in Reference [12].

Limits of the acceptance and resolution of the detector and effects due to initial state photon radiation were corrected by a bin-by-bin correction procedure. The correction factors were defined by the ratio of the distribution calculated from events generated by JETSET 6.3 at *hadron level* over the same distribution at *detector level*. The *hadron level* distributions were obtained from JETSET 6.3 generator runs without detector simulation and without initial state radiation, using all particles with lifetimes $\tau > 3 \cdot 10^{-10}$ s. Events at *detector level* contained initial state photon radiation and a detailed simulation of the detector response, and were processed in the same way as the data.

The data distributions were further corrected for hadronisation effects by applying bin-by-bin correction factors derived from the distributions at *parton level* and at *hadron level*, where the parton level is given by the partons before hadronisation. In the upper part of Fig. 1, the hadronisation correction for the distributions of the $C$-parameter at $\sqrt{s} = 35$ and 44 GeV are shown. The correction factors for all event shapes are typically of the order 10 to 30% increasing towards the 2-jet region.

Systematic uncertainties of the corrected data distributions were investigated by modifying details of the event selection and of the correction procedure. For each variation the whole analysis was repeated and any deviation from the main result was considered as a systematic error. In general, the maximum deviation from the main result for each kind of variation was regarded as a symmetric systematic uncertainty. The main result was obtained using the default selection and correction procedure as described above.

To estimate experimental uncertainties, we considered either tracks or clusters for the measurement of the event shape distributions. Furthermore, the selection cuts for multihadronic events were either loosened or tightened as described in [2], in order to check the influence of background events on the measurement.

The impact of the hadronisation model of the JETSET 6.3 generator was studied by varying the values of several significant model parameters around their tuned values from Reference [13] used for our main result. The variations amount to the percentage of the one standard deviations found in Reference [14] from a parameter tuning of JETSET. In detail, effects due to parton shower, hadronisation parameters, and quark masses were considered, following the procedure in [2]. These uncertainties of the hadronisation corrections are shown for the $C$-parameter distributions at $\sqrt{s} = 35$ and 44 GeV as shaded band around the correction factors in the upper part of Figure 1.

# 4 Determination of $\alpha_s$ at $\sqrt{s} = 35$ and 44 GeV

The data values for the event shape distributions and the corresponding mean value of the $C$-parameter corrected to *hadron level* are listed in Table 1, where also the statistical errors and experimental systematic uncertainties are given. Our measurements of the jet broadening event shape distributions can be found in Reference [2].

After correcting for hadronisation, the event shape distributions were compared directly with the analytic QCD calculations. We determined the strong coupling constant $\alpha_s$ by $\chi^2$ fits to the event shape distributions of $C$, $B_T$, and $B_W$. The fits to the $C$-parameter distributions considered the resummation results obtained in [5]. For the fits to the jet broadening measures we used the improved calculation of Reference [4]. For the sake of direct comparison with other published results we chose the $\ln(R)$-matching scheme[15] to merge the $\mathcal{O}(\alpha_s^2)$[16] with the NLLA calculations. The renormalisation scale factor, $x_\mu \equiv \mu/\sqrt{s}$ was set to $x_\mu = 1$ for the main result.

The fit ranges for each observable were chosen to be the largest range for which the hadronisation uncertainties remained less than 10 to 15%, for which the $\chi^2$/d.o.f. of the fits did not substantially exceed unity, and for which stable fits were obtained for changes of the fit range. The remaining changes in $\alpha_s$ when modifying the fit range by one bin on either side were taken as systematic uncertainties. Only statistical errors were considered in the fit.

In order to investigate the importance of higher order terms in the theory, we also changed the renormalisation scale factor in the range of $x_\mu = 0.5$ to 2.0. The associated renormalisation scale uncertainties are larger than those from the detector correction and the hadronisation model dependence.

Our results for $\alpha_s$ from $B_T$, $B_W$ and the $C$-parameter at $\sqrt{s} = 35$ and 44 GeV are given in Table 2, where also the statistical and systematic uncertainties of the measurements are given. It should be pointed out that the improved perturbative calculation for the jet broadening resulted in an increased $\alpha_s$ value, not affecting the systematic uncertainties but improving the $\chi^2$/d.o.f.

In Figure 2 the $\alpha_s$ values from this study are shown with those obtained from the distributions of thrust, heavy jet mass and the differential 2-jet rate using the Durham scheme as presented in our previous publication [2]. Replacing the $\alpha_s$ values obtained from the jet broadening observables, a single $\alpha_s$ value was obtained from the individual determinations from the six event shape observables following the procedure described in References [17]. This procedure accounts for correlations of the systematic uncertainties. At each energy, a weighted average of the six $\alpha_s$ values was calculated with the reciprocal of the respective squared total error used as a weight. For each of the systematic checks, the mean of the $\alpha_s$ values from all considered observables was determined. Any deviation of this mean from the weighted average of the main result was taken as a systematic uncertainty.

The final results for $\alpha_s$ are

$$\begin{aligned}\alpha_s(35\ \mathrm{GeV}) &= 0.1448 \pm 0.0010(\mathrm{stat.})\ ^{+0.0117}_{-0.0069}(\mathrm{syst.})\\ \alpha_s(44\ \mathrm{GeV}) &= 0.1392 \pm 0.0017(\mathrm{stat.})\ ^{+0.0104}_{-0.0072}(\mathrm{syst.})\ ,\end{aligned}$$

where the systematic errors at 35 and 44 GeV are the quadratic sums of the experimental uncertainties ($\pm 0.0017$, $\pm 0.0032$), the effects due to the Monte Carlo modelling ($^{+0.0070}_{-0.0035}$, $^{+0.0050}_{-0.0027}$) and the contributions due to the variation of the renormalisation scale ($^{+0.0092}_{-0.0057}$, $^{+0.0086}_{-0.0058}$). The modelling uncertainties due to quark mass effects contribute significantly to the total error.

# 5 Power corrections to the mean values of event shapes

The strong coupling constant $\alpha_s$ can also be extracted from the energy dependence of the mean values of event shape distributions. Non-perturbative effects are inherent in the event shape observables due to the contributions of very low energetic gluons and of hadronisation. For an observable $\mathcal{F}$ the effect of the gluons can be represented by additive power-suppressed corrections ($\langle \mathcal{F}^{\mathrm{pow.}} \rangle \propto 1/\sqrt{s}$) to the perturbative predictions of the mean values of the event shape distributions ($\langle \mathcal{F}^{\mathrm{pert.}} \rangle$). In the calculations of Reference [6, 18] which we used in this analysis a non-perturbative parameter

$$\bar{\alpha}_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} \mathrm{d}k \quad \alpha_s(k)$$

was introduced to replace the divergent portion of the perturbative expression for $\alpha_s(\sqrt{s})$ below an infrared matching scale $\mu_I$. The power corrections to event shape observables have been calculated in References [6] up to two-loops. It was found that the two-loop result is identical to the one-loop calculation of Reference [18] up to a factor $\mathcal{M} = 1.79 \pm 0.36$, which is known as the Milan factor, and a factor $2/\pi$, which is due to defining the non-perturbative parameter $\bar{\alpha}_0$ as above.

The power corrections for the mean values of thrust ($T$), heavy jet mass ($M_H^2/s$) and $C$-parameter is

$$\langle \mathcal{F}^{\mathrm{pow.}} \rangle = a_{\mathcal{F}} \cdot \frac{16}{3\pi} \frac{2\mathcal{M}}{\pi} \cdot \left( \frac{\mu_I}{\sqrt{s}} \right) \cdot \left( \bar{\alpha}_o(\mu_I) - \alpha_s(\sqrt{s}) + \mathcal{O}(\alpha_s^2) \right) \quad .$$

It was recently found in Reference [7] that an additional factor has to be applied in the case of jet broadening observables. In leading order the factors are given by

$$\frac{\pi}{4\sqrt{\alpha_s(0.472\sqrt{s})/3}} - 0.822 \quad , \quad \frac{\pi}{4\sqrt{2\alpha_s(0.472\sqrt{s})/3}} - 0.343$$

for total ($B_T$) and wide ($B_W$) jet broadening, respectively. The $a_{\mathcal{F}}$ coefficients which depend on the observable $\mathcal{F}$ are calculable. They assume the values $-1$, $1/2$, $1/2$, $1/4$, $3\pi/2$ for thrust, heavy jet mass, total and wide jet broadening, and $C$-parameter, respectively [6, 7].

The power correction to the mean values of an event shape $\mathcal{F}$ is an additive term to the perturbative prediction, $\langle\mathcal{F}\rangle = \langle\mathcal{F}^{\text{pert.}}\rangle + \langle\mathcal{F}^{\text{pow.}}\rangle$. We determined $\alpha_s(M_{\mathrm{Z}^0})$ and the non-perturbative parameter $\bar{\alpha}_0(\mu_I)$ from $\chi^2$ fits of this expression to the mean values of thrust, heavy jet mass, $C$-parameter, total and wide jet broadening. The fits included the measured mean values published by various experiments at different centre-of-mass energies [2, 19] between 14 and 183 GeV. For the central fit results of $\alpha_s$, we fixed the renormalisation scale factor $x_\mu$ to one and the infrared scale to $\mu_I = 2$ GeV. Theoretical systematic uncertainties were assessed by varying $x_\mu$ from 0.5 to 2, $\mu_I$ from 1 to 3 GeV, and $\mathcal{M}$ within the quoted uncertainty.

Figure 3 shows the data values and the fit curves for the $C$-parameter, the total and the wide jet broadening, $B_T$ and $B_W$. The $\chi^2$/d.o.f. of the fits is between 0.7 (for the heavy jet mass) and 1.3 (for the thrust). The numeric results of all fits are summarized in Table 3, presenting the values for $\alpha_s(M_{\mathrm{Z}^0})$ and for $\bar{\alpha}_0(\mu_I)$, their experimental errors and systematic uncertainties. We also quote averages of the individual $\alpha_s$ and $\bar{\alpha}_0$ results which were calculated according to the procedure used in Section 4. We consider these results as a test of the new theoretical prediction [6, 7] for the power corrections to the mean values of event shapes. It should be noted that the individual results for $\bar{\alpha}_0(\mu_I)$ scatter around the average value of $\bar{\alpha}_0(2\text{ GeV}) = 0.473^{+0.058}_{-0.041}$, where the error is the total uncertainty. The r.m.s. of the scatter is 0.076. The theoretically expected universality of $\bar{\alpha}_0$ is, therefore, observed at the level of better than 20%.

Our final value of the strong coupling from the fits to the energy dependence of the mean values of event shapes is

$$\alpha_s(M_{\mathrm{Z}^0}) = 0.1177\ ^{+0.0035}_{-0.0034} \quad ,$$

where the error is the total uncertainty. This result lies about 2% above the value obtained in our previous analysis [2], which did not consider the new Milan correction nor the revisited calculation for the power correction of the jet broadening observables. Our updated result is in good agreement with the world average value [20] of $\alpha_s^{\text{w.a.}}(M_{\mathrm{Z}^0}) = 0.119 \pm 0.004$ and is of similar precision.

# 6 Summary and conclusions

Multihadronic data recorded by the JADE experiment were used to measure $C$-parameter distributions at centre-of-mass energies of $\sqrt{s} = 35$ and 44 GeV for the first time.

We updated our determinations [2] of the strong coupling constant $\alpha_s$ at $\sqrt{s} = 35$ and 44 GeV using $\ln(R)$-matching for the combination of $\mathcal{O}(\alpha_s^2)$ and resummed calculations now including the $C$-parameter. Improved perturbative calculations to the jet broadening measures $B_T$ and $B_W$ were applied. We found these calculations to describe the data better. The $\alpha_s$ values were also found to be more consistent with those from other event shape observables.

Evolving our combined $\alpha_s$ measurements to $\sqrt{s} = M_{\mathrm{Z}^0}$ the results obtained at 35 and 44 GeV are $0.123\,^{+0.008}_{-0.005}$ and $0.123\,^{+0.008}_{-0.006}$, respectively. Their combination yields $\alpha_s(M_{\mathrm{Z}^0}) = 0.123\,^{+0.008}_{-0.005}$.

We further investigated mean values of the event shape distributions for the energy range of the PETRA and LEP colliders and compared directly with analytic QCD predictions plus power corrections for hadronisation effects, the latter involving an universal non-perturbative parameter $\bar{\alpha}_0$. Our present studies, based on two-loop calculations for the event shapes, which lead to the Milan correction factor, yield

$$\alpha_s(M_{\mathrm{Z}^0}) = 0.1177\,^{+0.0035}_{-0.0034} \quad ,$$

which is in good agreement with our results from the $\mathcal{O}(\alpha_s^2)$+NLLA fits and also with the world average value. The predicted universality of the non-perturbative parameter $\bar{\alpha}_0$ is found at a level of better than 20%. We point out that the data points provided by JADE did significantly reduce the statistical uncertainties of the fits of the predictions to the mean values of the distributions, in particular for the $C$-parameter.

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# Tables

| $C$ | $1/\sigma \cdot \mathrm{d}\sigma/\mathrm{d}C$ | |
|---|---|---|
| | $\sqrt{s} = 35$ GeV | $\sqrt{s} = 44$ GeV |
| 0.00-0.08 | $0.064 \pm 0.004 \pm 0.034$ | $0.093 \pm 0.008 \pm 0.031$ |
| 0.08-0.12 | $0.434 \pm 0.019 \pm 0.079$ | $0.801 \pm 0.047 \pm 0.131$ |
| 0.12-0.16 | $1.383 \pm 0.039 \pm 0.068$ | $1.953 \pm 0.086 \pm 0.250$ |
| 0.16-0.20 | $2.436 \pm 0.056 \pm 0.068$ | $3.016 \pm 0.116 \pm 0.189$ |
| 0.20-0.24 | $2.678 \pm 0.060 \pm 0.229$ | $3.172 \pm 0.123 \pm 0.288$ |
| 0.24-0.28 | $2.845 \pm 0.062 \pm 0.260$ | $2.759 \pm 0.116 \pm 0.141$ |
| 0.28-0.32 | $2.289 \pm 0.054 \pm 0.208$ | $2.457 \pm 0.108 \pm 0.286$ |
| 0.32-0.36 | $2.327 \pm 0.056 \pm 0.269$ | $1.748 \pm 0.086 \pm 0.098$ |
| 0.36-0.40 | $1.837 \pm 0.047 \pm 0.109$ | $1.623 \pm 0.082 \pm 0.230$ |
| 0.40-0.44 | $1.441 \pm 0.041 \pm 0.131$ | $1.163 \pm 0.068 \pm 0.150$ |
| 0.44-0.48 | $1.254 \pm 0.038 \pm 0.066$ | $1.159 \pm 0.070 \pm 0.210$ |
| 0.48-0.52 | $1.024 \pm 0.034 \pm 0.071$ | $0.847 \pm 0.056 \pm 0.071$ |
| 0.52-0.58 | $0.839 \pm 0.025 \pm 0.034$ | $0.720 \pm 0.041 \pm 0.070$ |
| 0.58-0.64 | $0.702 \pm 0.022 \pm 0.095$ | $0.626 \pm 0.038 \pm 0.046$ |
| 0.64-0.72 | $0.532 \pm 0.017 \pm 0.030$ | $0.503 \pm 0.030 \pm 0.050$ |
| 0.72-0.82 | $0.453 \pm 0.014 \pm 0.051$ | $0.348 \pm 0.022 \pm 0.034$ |
| 0.82-1.00 | $0.090 \pm 0.004 \pm 0.010$ | $0.079 \pm 0.008 \pm 0.015$ |
| mean value | $0.3673 \pm 0.0013 \pm 0.0040$ | $0.3404 \pm 0.0023 \pm 0.0037$ |

Table 1: Event shape data at $\sqrt{s} = 35$ (left) and 44 GeV (right) for the $C$-parameter observable. The values were corrected for detector and for initial state radiation effects. The first error denotes the statistical and the second the experimental systematic uncertainty.

| $\sqrt{s}$ | Observable | $\alpha_s(\sqrt{s})$ | stat. | exp. | MC | renorm. |
|---|---|---|---|---|---|---|
| 35 GeV | $B_T$ | 0.1489 | $\pm 0.0008$ | $\pm 0.0014$ | $^{+0.0099}_{-0.0048}$ | $^{+0.0136}_{-0.0107}$ |
| | $B_W$ | 0.1367 | $\pm 0.0009$ | $\pm 0.0026$ | $^{+0.0089}_{-0.0045}$ | $^{+0.0096}_{-0.0075}$ |
| | $C$ | 0.1480 | $\pm 0.0009$ | $\pm 0.0014$ | $^{+0.0097}_{-0.0058}$ | $^{+0.0138}_{-0.0110}$ |
| 44 GeV | $B_T$ | 0.1458 | $\pm 0.0014$ | $\pm 0.0037$ | $^{+0.0072}_{-0.0032}$ | $^{+0.0127}_{-0.0100}$ |
| | $B_W$ | 0.1318 | $\pm 0.0016$ | $\pm 0.0051$ | $^{+0.0054}_{-0.0026}$ | $^{+0.0081}_{-0.0061}$ |
| | $C$ | 0.1470 | $\pm 0.0017$ | $\pm 0.0027$ | $^{+0.0073}_{-0.0044}$ | $^{+0.0133}_{-0.0107}$ |

Table 2: Values of $\alpha_s$ obtained from fits of $\mathcal{O}(\alpha_s^2)$ +NLLA QCD calculations to the distributions of total and wide jet broadening, $B_T$ and $B_W$, and $C$-parameter, at $\sqrt{s} =$ 35 and 44 GeV. Additionally, the statistical (stat.) and the experimental errors (exp.) of the fit, the uncertainties due to the Monte Carlo modelling (MC) of the hadronisation and due to the choice of the renormalisation scale (renorm.) are given.

| (a) | $\langle 1-T \rangle$ | $\langle M_H^2/s \rangle$ | $\langle B_T \rangle$ | $\langle B_W \rangle$ | $\langle C \rangle$ | average |
|---|---|---|---|---|---|---|
| $\alpha_S(M_{Z^0})$ | **0.1198** | **0.1141** | **0.1183** | **0.1190** | **0.1176** | **0.1177** |
| $Q$ range [GeV] | 13-183 | 14-183 | 35-183 | 35-183 | 35-183 | |
| $\chi^2$/d.o.f. | 52.2/39 | 22.0/33 | 22.1/25 | 18.8/26 | 18.8/16 | |
| experimental | $\pm 0.0013$ | $\pm 0.0010$ | $\pm 0.0016$ | $\pm 0.0020$ | $\pm 0.0013$ | $\pm 0.0016$ |
| $x_\mu = 0.5$ | $-0.0049$ | $-0.0026$ | $-0.0038$ | $+0.0017$ | $-0.0043$ | $-0.0027$ |
| $x_\mu = 2.0$ | $+0.0061$ | $+0.0037$ | $+0.0048$ | $+0.0003$ | $+0.0053$ | $+0.0026$ |
| $\mathcal{M} - 20\%$ | $+0.0011$ | $+0.0013$ | $+0.0008$ | $+0.0005$ | $+0.0009$ | $+0.0008$ |
| $\mathcal{M} + 20\%$ | $-0.0011$ | $-0.0001$ | $-0.0007$ | $-0.0005$ | $-0.0009$ | $-0.0005$ |
| $\mu_I = 1$ GeV | $+0.0025$ | $+0.0013$ | $+0.0017$ | $+0.0011$ | $+0.0020$ | $+0.0014$ |
| $\mu_I = 3$ GeV | $-0.0019$ | $-0.0011$ | $-0.0014$ | $-0.0009$ | $-0.0016$ | $-0.0012$ |
| Total error | $^{+0.0068}_{-0.0055}$ | $^{+0.0043}_{-0.0030}$ | $^{+0.0054}_{-0.0044}$ | $^{+0.0029}_{-0.0022}$ | $^{+0.0058}_{-0.0049}$ | $^{+0.0035}_{-0.0034}$ |

| (b) | $\langle 1-T \rangle$ | $\langle M_H^2/s \rangle$ | $\langle B_T \rangle$ | $\langle B_W \rangle$ | $\langle C \rangle$ | average |
|---|---|---|---|---|---|---|
| $\bar{\alpha}_0$(2 GeV) | **0.509** | **0.614** | **0.442** | **0.392** | **0.451** | **0.473** |
| experimental | $\pm 0.012$ | $\pm 0.018$ | $\pm 0.015$ | $\pm 0.028$ | $\pm 0.010$ | $\pm 0.014$ |
| $x_\mu = 0.5$ | $+0.003$ | $+0.011$ | $+0.020$ | $+0.109$ | $+0.005$ | $+0.018$ |
| $x_\mu = 2.0$ | $-0.002$ | $-0.005$ | $-0.014$ | $-0.042$ | $-0.003$ | $-0.009$ |
| $\mathcal{M} - 20\%$ | $+0.058$ | $+0.084$ | $+0.046$ | $+0.032$ | $+0.050$ | $+0.053$ |
| $\mathcal{M} + 20\%$ | $-0.040$ | $-0.064$ | $-0.031$ | $-0.022$ | $-0.034$ | $-0.037$ |
| Total error | $^{+0.059}_{-0.042}$ | $^{+0.087}_{-0.067}$ | $^{+0.052}_{-0.037}$ | $^{+0.117}_{-0.055}$ | $^{+0.051}_{-0.036}$ | $^{+0.058}_{-0.041}$ |

Table 3: Values of $\alpha_s(M_{Z^0})$ (a) and $\bar{\alpha}_0(\mu_I)$ (b) derived using $\mu_I = 2$ GeV and $x_\mu = 1$ and the $\mathcal{O}(\alpha_s^2)$ calculations plus two-loop power corrections, which include the Milan factor [6], and the revisited power corrections for the broadening observables [7]. In addition, the statistical and systematic uncertainties are given. Signed values indicate the direction in which $\alpha_s(M_{Z^0})$ and $\bar{\alpha}_0(\mu_I)$ changed with respect to the standard analysis. The renormalisation and infrared scale uncertainties are treated as an asymmetric uncertainty on $\alpha_s(M_{Z^0})$ and $\bar{\alpha}_0(\mu_I)$. No error contribution from the infrared scale $\mu_I$ is assigned to $\bar{\alpha}_0(\mu_I)$.

# Figures

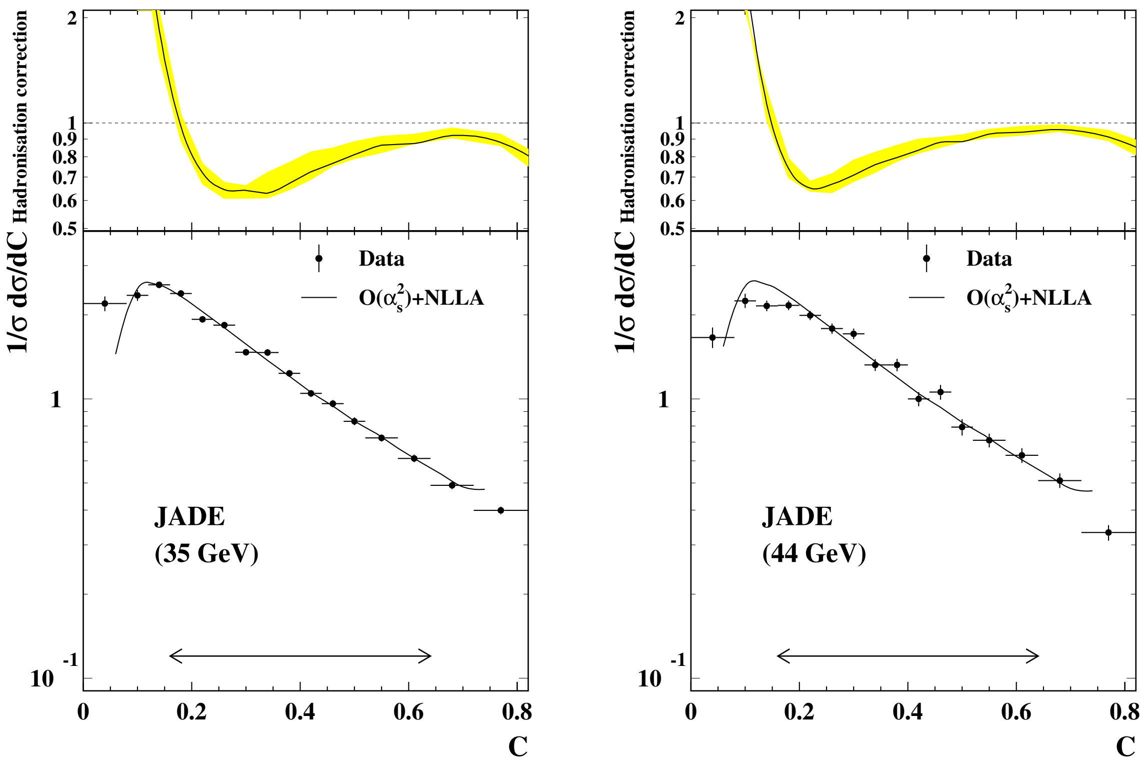


Figure 1: Distributions for $C$-parameter (bottom) measured at $\sqrt{s}$ = 35 and 44 GeV and corrected to *parton level*. The fits of the new $\mathcal{O}(\alpha_s^2)$+NLLA QCD calculation for the $C$-parameter are overlaid and the fit ranges are indicated by the arrows. The error bars represent statistical errors only. Also shown are the hadronisation correction factors (top). The shaded bands represent the uncertainties due to the modelling of the parton shower and string fragmention by JETSET 6.3.

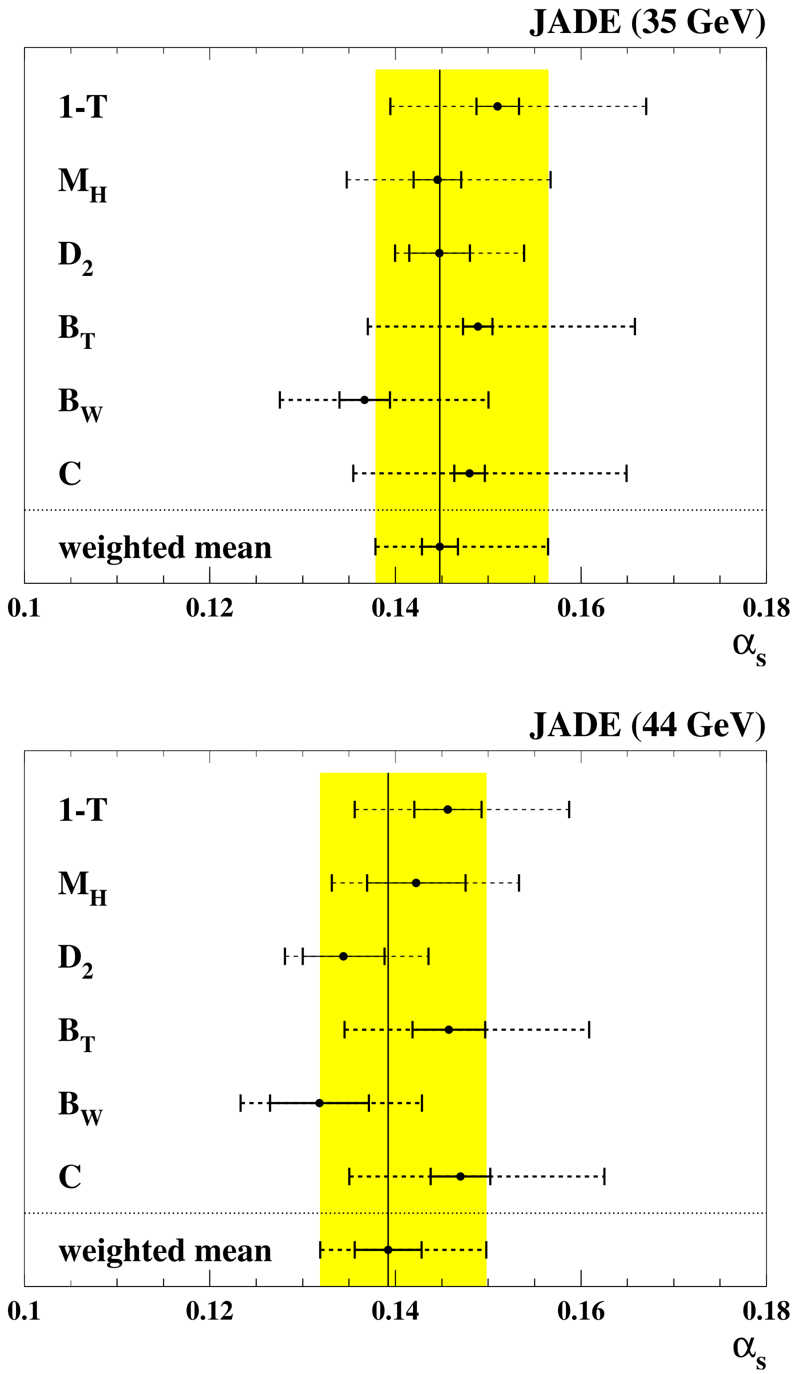


Figure 2: Values of $\alpha_s(35\text{ GeV})$ and $\alpha_s(44\text{ GeV})$ derived from $\mathcal{O}(\alpha_s^2)$+NLLA fits to event shape distributions. The experimental and statistical uncertainties are represented by the solid error bars. The dashed error bars show the total error including hadronisation and higher order effects. The shaded region shows the one standard deviation region around the weighted average (see text).

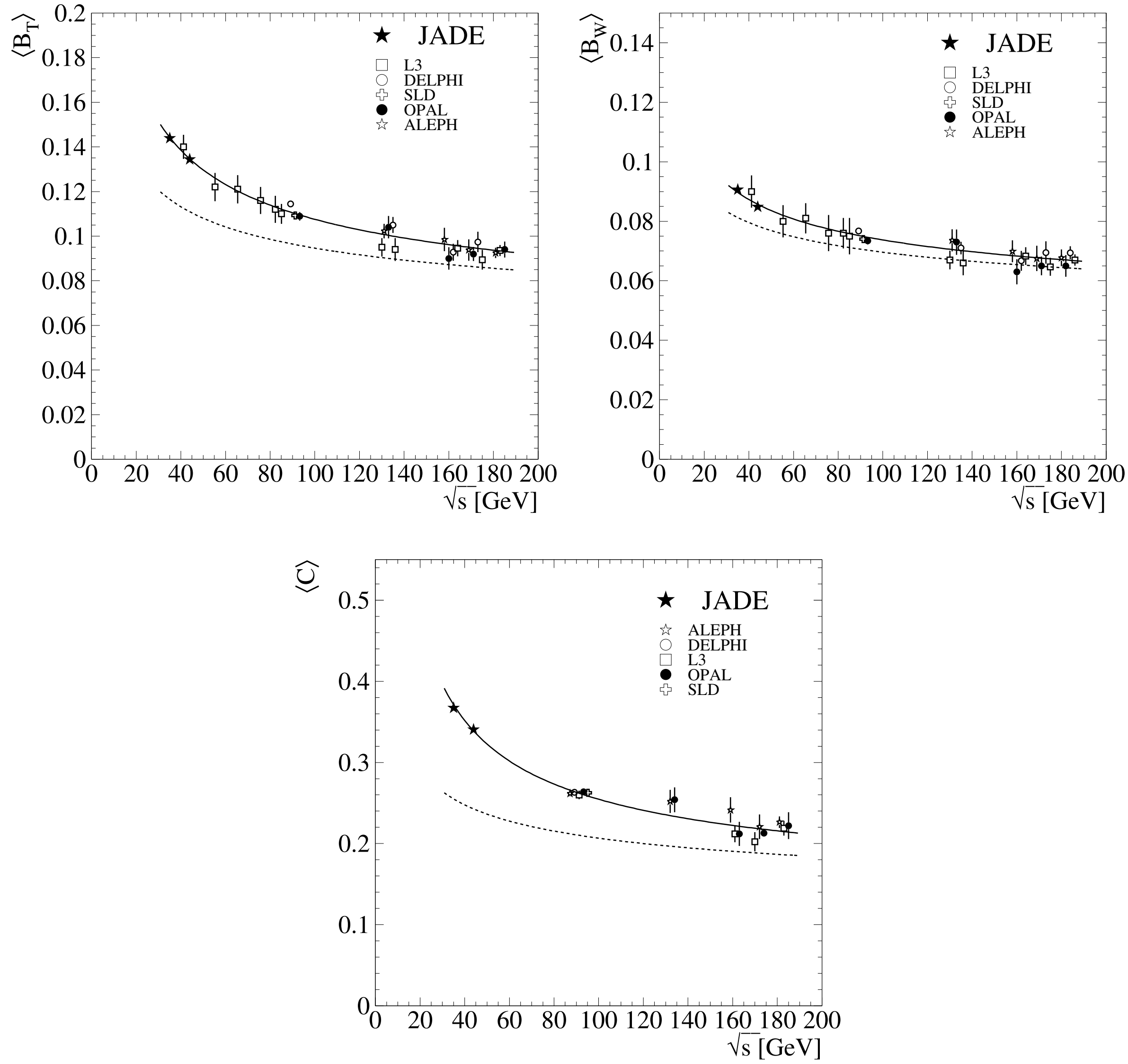


Figure 3: Energy dependence of the mean values of the total ($\langle B_T \rangle$) and wide jet broadening ($\langle B_W \rangle$), and of the $C$-parameter ($\langle C \rangle$) are shown [19]. The solid curves are the result of the fit using perturbative calculations plus two-loop power corrections which include the Milan factor [6] and the revisited power corrections to jet broadening observables[7]. The dashed line is the perturbative prediction using the fitted value of $\alpha_s(M_{Z^0})$.