December 5, 2001
Version 1.0

Homework #10 (due on Dec. 7 Friday)

CP Violation Measurements in the $B^o - \bar{B}^o$system (sin2β)

1. The CDF experiment has measured sin$2\beta$to be 0.79 ± 0.42, and BaBar and Belle experiments 0.59 ± 0.14 (statistical)±0.05 (systematic) and 0.99 ± 0.14 (statistical)$\pm$ 0.06 (systematic), respectively.

   Assuming that there be no correlated errors among the measurements, combine the BaBar and Belle measurements.

2. Does the fact that the central value of the CDF result is the same as the BaBar-Belle combined measurement indicate

   (a) CDF is lucky, or

   (b) CDF got the right answer ?

   Explain your answer quantitatively (calculate the probability).

3. If the Belle and BaBar measurements have a correlated error of 0.1 (this is not a true statement, but just for practice), what will be the total uncertainty of their combined measurement ?

4. The total luminosity used for the BaBar measurement was about 30 fb^{-1} (a similar number for the Belle measurement). If the BaBar experiment accumulates another 30 fb^{-1} by Summer 2002, what will be their total uncertainty on sin2β? Assume that the systematic error does not change with a larger statistics although we expect the systematic error will get smaller with statistics by studying the error with large control samples. *Hint* : the statistical error goes like $1/\sqrt{N_{events}}$.

Neutrino Oscillations

1. Let us follow the life of a neutrino as it traverses the Sun. An electron neutrino, ν_e, born at $t = 0$ in the core of the Sun (solar neutrino) is described by

$$|\nu_e(t) >= \cos\theta\ |\nu_1(t=0) > +\sin\theta\ |\nu_2(t=0) >$$

where $|\nu_1>$ and $|\nu_1>$ are the mass eigenstates. Show that the probability of finding ν_μ after traveling a distance $L = t$ is

$$P_{\nu_e \to \nu_\mu}(t) = \sin^2 2\theta \, \sin^2(\frac{\Delta m^2 \, L}{4E})$$

where $E = p$ is the neutrino momentum (assuming a small neutrino mass) and θ is the mixing angle between the two neutrino states.

Here we ignored the matter effects: the combinations that are eigenstates in free space will not remain eigenstates when passing through matter because while neutrinos of all flavours can have neutral current interactions with the electrons of the medium via a Z boson exchange, only electron neutrinos can interact with the electrons of the medium via a W boson exchange in addition to the Z exchange.

2. With E measured in GeV, L in km, and Δm^2 in eV^2, show the previous probability can be written as

$$P_{\nu_e \to \nu_\mu}(t) = \sin^2 2\theta \, \sin^2(1.27 \, \frac{\Delta m^2 \, L}{E}).$$

Note that this formula can be applied to any two-flavor neutrino oscillations.

3. The Super-K (Super-Kamiokande) experiment has analysed atmospheric neutrino (ν_μ and ν_e) data and showed that the data are consistent with two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with $\sin^2 2\theta \simeq 1$ and $\Delta m^2 \simeq 10^{-3}$ eV^2.

They've also looked at the ratio of the number of data to Monte Carlo ν_μ events for flavor changing charged current (by identifying muons in the events) and the (approximated) results are shown below. Here, Monte Carlo assumed no neutrino oscillations.

L/E_ν (km / GeV)	Data / Monte Carlo
0.7	1.00 ± 0.20
20	1.00 ± 0.08
70	0.90 ± 0.08
200	0.83 ± 0.15
700	0.65 ± 0.15
2000	0.70 ± 0.08
7000	0.63 ± 0.06
20000	0.61 ± 0.08

Make a figure with the results above using the horizontal axis of L/E_ν in log scale between 1 and 10^5 km/GeV and the vertical scale of the ratio in linear scale between 0 and 1.5. At large L/E_ν, the ν_μ have presumably undergone numerous oscillations and have average out to roughly half the initial rate.

Using $\sin^2 2\theta = 1$, make a rough estimate of Δm^2.