Version 1.0

Homework #8 Solutions

Top Quark

Question #1

$$\begin{aligned} M_W^2 &= (P_\mu + P_\nu)^2 = {P_\mu}^2 + {P_\nu}^2 + 2P_\mu P_\nu \\ &= M_\mu^2 + M_\nu^2 + 2(E_\mu E_\nu - \vec{P_\mu} \cdot \vec{P_\nu}) \\ &\simeq 2(E_\mu E_\nu - \vec{P_\mu^+} \cdot \vec{P_\nu^+} - P_\mu^{||} P_\nu^{||}) \\ &= 2(P_\mu \sqrt{{P_\nu^+}^2 + {P_\nu^{||}}^2} - \vec{P_\mu^+} \cdot \vec{P_\nu^+} - P_\mu^{||} P_\nu^{||}) \\ P_\nu &= \frac{{P_\mu}^2}{{P_\mu^{||}}^2} \cdot (\frac{M_W^2}{2} + \vec{P_\mu^+} \cdot \vec{P_\nu^+}) \pm \frac{P_\mu}{{P_\mu^+}^2} \cdot \sqrt{(\frac{M_W^2}{2} + P_\mu^+ P_\nu^+ cos\phi)^2 - {P_\mu^+}^2 {P_\nu^+}^2} \end{aligned} \tag{0.1}$$

No solutions when $(\frac{M_W^2}{2} + P_\mu^+ P_\nu^+ cos\phi)^2 - {P_\mu^+}^2 {P_\nu^+}^2 < 0$.
That is,

$$\begin{aligned} (\frac{M_W^2}{2} + P_\mu^+ P_\nu^+ cos\phi)^2 &< {P_\mu^+}^2 {P_\nu^+}^2 \\ \frac{M_W^4}{4} + M_W^2 P_\mu^+ P_\nu^+ cos\phi + {P_\mu^+}^2 {P_\nu^+}^2 cos^2\phi &< {P_\mu^+}^2 {P_\nu^+}^2 \\ \frac{M_W^4}{4} + M_W^2 P_\mu^+ P_\nu^+ cos\phi + {P_\mu^+}^2 {P_\nu^+}^2 \ (1 - cos^2\phi) &< 0 \\ cos^2\theta + cos\theta \ \frac{M_W^2}{P_\mu^+ P_\nu^+} + \frac{M_W^4}{4{P_\mu^+}^2 {P_\nu^+}^2} - 1 &< 0 \\ cos\theta_\pm = -\frac{M_W^2}{2P_\mu^+ P_\nu^+} \pm 1 & \end{aligned} \tag{0.2}$$

No solutions if

$$\begin{aligned} -\frac{M_W^2}{2P_\mu^+ P_\nu^+} - 1 < cos\theta < -\frac{M_W^2}{2P_\mu^+ P_\nu^+} + 1 \\ -1 < cos\theta < -\frac{M_W^2}{2P_\mu^+ P_\nu^+} + 1 \end{aligned} \tag{0.3}$$