11

The Fifth Quark

Discovery of the Υ and the B meson, 1977–1987

The discovery of the J/ψ and charmed quark seemed to complete a family of fermions, (c, s, ν_μ, μ), entirely analogous to (u, d, ν_e, e). If this pattern were indicative, then the τ and its neutrino presaged a new pair of quarks. Both e^+e^- annihilation and hadronic production of lepton pairs, the techniques that had uncovered the charmed quark, were extended in the search for the next quark.

Leon Lederman and his co-workers **(Ref. 11.1)** pressed the search for peaks in the $\mu^+\mu^-$ spectrum to high energies by studying the collisions of 400 GeV protons on nuclear targets at Fermilab. Their apparatus was a double-arm spectrometer set to measure $\mu^+\mu^-$ pairs with invariant masses above 5 GeV with a resolution of 2%. Hadrons were eliminated by using long beryllium filters in each arm. In mid-1977, a clear, statistically significant $\mu^+\mu^-$ peak was observed in the 9.5 GeV region with an observed width of about 1.2 GeV. A more detailed analysis showed better agreement with two peaks at 9.44 and 10.17 GeV, which were given the names Υ and Υ'. It soon became evident that this was a repetition of the J/ψ and ψ' story.

With the help of an energy upgrade, in May 1978 two groups at the DORIS e^+e^- storage ring at DESY were able to observe the Υ in the PLUTO and DASP II detectors. The results of the experiments are reproduced here (ref **11.2**, 11.3) and in Figure 11.1. The determination of the mass of the resonance was greatly improved with the result $M_\Upsilon = 9.46 \pm 0.01$ GeV. Moreover, the observed width was limited only by the energy spread of the beams, so that it was less than 1/100 as much as that observed in hadronic production. Just as for the J/ψ, it was possible to infer the partial width for $\Upsilon \to e^+e^-$ from the area under the resonance curve, with the result $\Gamma_{e^+e^-}(\Upsilon) = 1.3 \pm 0.4$ keV. Using nonrelativistic potential models derived from the ψ system and the assumption that the potential was independent of the quark type, it was possible to predict the wave function at the origin and thus $\Gamma_{e^+e^-}(\Upsilon)$ for the cases of charge $-1/3$ and $+2/3$. The comparison indicated that the new quark had charge $-1/3$ rather than $+2/3$. The new quark was dubbed

Figure 11.1: Measurements of the e^+e^- cross section at the lower Υ states. Measurements from the DASP II experiment at DORIS show (a) the Υ and (b) the Υ' (Ref. 11.3). Measurements by the CLEO group at CESR show (c) the Υ, (d) the Υ', and (e) the Υ''. Discrepancies between the mass measurements by the two groups were later resolved (Ref. 11.5).

the b for "bottom," reflecting the practice of writing the quark pairs (u, d) and (c, s) with the charge $-1/3$ beneath the charge $2/3$ quark. Thus the prospective sixth quark was referred to as t or "top".

After additional cavities were added to increase the energy of the DORIS ring, the DESY–Heidelberg lead-glass detector and DASP II were able to observe the Υ' (Refs. 11.3, **11.4**). The $\Upsilon' - \Upsilon$ splitting was found to be very nearly the same as that for $\psi' - \psi$.

By 1980, the Cornell Electron Storage Ring (CESR) with its two detectors, CLEO and CUSB, became operational. They both observed the Υ and Υ', and additional resonances, Υ'' and Υ''' (Refs. 11.5, 11.6, 11.7, **11.8**). The first three states, with masses 9.460, 10.023, and 10.355 GeV, are narrow, with observed widths consistent with the beam spread of the machine. They are analogous to the ψ and ψ', and correspond to $1^3S_1, 2^3S_1$, and 3^3S_1 states of a $b\overline{b}$ system. Figure 11.1 shows the Υ and Υ' as observed by DASP II and the Υ, Υ', and Υ'' as observed by CLEO. The Υ''' at 10.580 GeV is a broader state, like the $\psi(3772) = \psi''$, and is interpreted as the 4^3S_1 state, lying above the threshold for $B\overline{B}$ production, where B represents a meson containing a $\overline{b}$ quark and a u or d quark. Thus $B^+ = \overline{b}u, B^0 = \overline{b}d, B^- = b\overline{u}, \overline{B}^0 = b\overline{d}$.

The existence of a series of s-wave bound states required that there be p-wave states as well. These were observed through radiative transitions from the s-wave states, $\Upsilon' \to \chi_b\gamma$, where χ_b represents a $C = +1$, $P = +1$ p-wave state. Evidence was obtained from the inclusive photon spectrum, $\Upsilon' \to \gamma +$ anything, and from the cascade $\Upsilon' \to \gamma\chi_b$, $\chi_b \to \gamma\Upsilon$, $\Upsilon \to l^+l^-$, where l represents e or μ. Measurements were carried out by CUSB and CLEO at CESR and by the Crystal Ball at DORIS II after the detector was shipped from Stanford to Hamburg. In Figure 11.2 some results from the Crystal Ball are shown.

Actual identification of B meson decays promised to be a formidable task, even though some lessons had been learned from the study of charm. By focusing on the 4^3S_1 Υ''' it was possible to obtain a good sample of $\Upsilon''' \to B\overline{B}$ events **(Ref. 11.9)**. The technique used was to identify candidates for D^0s and D^{*+}s, using only entirely charged decay modes, and combine these with either one or two charged pions. In analogy with the fundamental decay $c \to s$ leading to $D^+ \to K^-\pi^+\pi^+$, the transition $b \to c$ produces $B^- \to D^+\pi^-\pi^-$, $D^{*+}\pi^-\pi^-$. The combinations $B \to D\pi$ and $B \to D\pi\pi$ were required to produce Bs with energy equal to the beam energy since the decay is $\Upsilon''' \to B\overline{B}$. An accumulation of events for mass near 5.275 GeV demonstrated the observation of exclusive B decays. The number of events observed for three of the four decay modes examined was much greater than would be expected from the now known branching ratios. This was probably the result of mistakenly including some events in which there were additional unobserved low-momentum particles contributing to the decay.

The CLEO Collaboration at CESR and the ARGUS Collaboration at DESY identified additional decay modes. Particularly noteworthy were $B^+ \to \psi K^+$ and

Figure 11.2: The photon spectrum from Υ' decays obtained by the Crystal Ball Collaboration at DORIS II. A triplet of lines corresponding to $\Upsilon' \to \gamma\chi_b(^3P_{2,1,0})$ is seen between 100 and 200 MeV. The decays $\chi_b \to \gamma\Upsilon$ produce the unresolved signal between 400 and 500 MeV [R. Nernst *et al.*, *Phys. Rev. Lett.* **54**, 2195 (1985)].

$B^+ \to \psi' K^+$ and the analogous neutral B decays.

The CUSB Collaboration observed photons of energy about 50 MeV associated with $B\overline{B}$ production at energies above the Υ'''. This they ascribed to the production of $B^*\overline{B}$ and the subsequent decay $B^* \to B\gamma$. The splitting between the spin-one B^* and the pseudoscalar B was determined to be $52 \pm 2 \pm 4$ MeV. Some of the CUSB data are shown in Figure 11.3.

Early measurements, between 1982 and 1984, of the lifetimes of B particles, starting with the MAC detector at PEP gave surprisingly large values. Several collaborations used semileptonic decays to obtain B-enriched samples of events. The Mark II Collaboration at PEP built a vertex detector using a precision drift chamber located close to the interaction point, which allowed measurements of the distance of closest approach of the lepton tracks to the beam–beam collision region. The experiment found a B lifetime of about 1 ps. Similar measurements were made by the TASSO and JADE experiment at PETRA and the DELCO experiment at PEP.

It was possible to make much more precise measurements once Bs could be produced and detected at higher energies. The decays of Zs (see Chapter 13) are an especially useful source and lifetimes were measured at both CERN's Large Electron-Positron Collider (LEP) and at the SLAC Linear Detector (SLD), both of which are described in Chapter 13. Other high precision measurements were

Figure 11.3: The photon energy spectrum obtained by the CUSB Collaboration for events with high energy leptons and thrust less than 0.88 (indicative of events more spherical than the ordinary two-jet events produced in e^+e^- annihilation). These criteria signal the presence of B mesons. The e^+e^- c.m. energy for the solid histogram in (a) is 10.62 – 11.25 GeV, above the $B^*\overline{B}$ threshold. The dotted histogram in (a) was taken at the Υ''', below the $B^*\overline{B}$ threshold. In (b) the spectrum with the background subtracted shows a line near 50 MeV, ascribed to $B^* \to B\gamma$ (K. Han *et al.*, *Phys. Rev. Lett.* **55**, 36 (1985)).

made by CDF at Fermilab's Tevatron Collider. The lifetime found for the charged B meson was 1.65 ± 0.04 ps, while that for the neutral B mesons was 1.56 ± 0.04 ps.

In addition to the $\overline{b}d = B_d^0$ and $\overline{b}u = B^+$ mesons, it was natural to expect $\overline{b}s = B_s^0$ and $\overline{b}c = B_c^+$. Similarly, baryons containing b quarks were anticipated. It was equally natural to expect that all these would be produced less often and would thus be hard to reconstruct completely. There were, however, means of finding circumstantial evidence for these particles.

The CUSB-II Collaboration at Cornell's CESR studied the photons with energies 30 - 70 MeV produced at the $\Upsilon(5S)$ resonance. Some of these were due to the production of B^*. The energies of these photons were spread by Doppler broadening. In 1990 they reported that the observed spectrum was not described simply by a combination of BB, BB^*, and B^*B^* events. This would have given a broader distribution than they observed. From this they inferred the presence of the more massive B_s^*. The B_s^* would decay $B_s^* \to B_s\gamma$. The B_s^* would be produced with less velocity than the ordinary B^*, resulting less Doppler broadening.

Semileptonic decays could provide a more specific signature for B_s. At the quark level, these would give $\overline{b}s \to \overline{c}s\ell\nu$. Thus the signal sought was a lepton together with a D_s. The latter decays non-leptonically as $c\overline{s} \to s\overline{s}u\overline{d}$ which is most easily seen as $\phi\pi^+$ or $K^{*0}K^+$. In this way, the DELPHI, ALEPH, and OPAL collaborations working at LEP all found strong evidence for the existence of the B_s in 1992. The presence of a neutrino in the final state precluded complete reconstruction of the B_s and determination of its mass.

The dominant non-leptonic decays of the B_s are described at the quark level by $\overline{b}s \to \overline{c}(c\overline{s})s$ and $\overline{b}s \to \overline{c}(u\overline{d})s$. The former can produce $B_s \to J/\psi\phi$, quite analogous to $B_d \to J/\psi K$, while the latter gives $D_s^-\pi^+$. Because the J/ψ and ϕ are very narrow states with extremely clear signatures, $B_s \to J/\psi\phi$ provides the most effective way to determine the mass of the B_s. A candidate event of this sort found by OPAL had an invariant mass of 5.36 ± 0.07 GeV. A later re-analysis of the event showed the mass to be $5.359 \pm 0.019 \pm 0.007$ GeV, where the first error is due to measurement uncertainties and the latter to systematic effects. ALEPH found a $\psi'\phi$ event in which the ψ' decayed to $\mu^+\mu^-$. The mass of the reconstructed B_s was found to be $5.3684 \pm 0.0056 \pm 0.0015$ GeV. The DELPHI Collaboration found one event in each of three modes, $D_s^-\pi^+$, $D_s a_1^+$, and $J/\psi\phi$. The D_s was detected through its decay to $\phi\pi$. Combining the three resulted in a mass measurement of $5.374 \pm 0.016 \pm 0.002$ GeV.

It was natural for CESR operating at the $\Upsilon(4s)$ and LEP at the Z^0 to study B mesons. At both these machines a large percentage of the events have a $B\overline{B}$ pair. Additionally, the multiplicity of each event is low. It was, however, somewhat of a surprise that CDF at the Fermilab proton-proton Tevatron Collider was able to compete in B physics. To overcome the disadvantage of the relative rarity of B meson production in hadronic collisions, CDF relied on the extremely effective signative provided by B decays to states containing a J/ψ and on high-precision tracking.

in 1991, CDF installed a silicon vertex detector (SVX). While silicon detectors with fine spatial resolution had been used in fixed target experiments and at e^+e^- colliders, their use at hadron colliders was novel. The SVX featured 40,000 channels of silicon strips, each about 60 μm wide. An application-specific integrated circuit was developed to read out the data, suppressing all channels but those with interesting data. The spatial resolution of the SVX in the plane perpendicular to the beam was of the order of 20 μm, precise enough to identify the separation of the decay vertices of B mesons from the collision point. The success of the SVX in B physics and subsequently in t physics meant that all future hadron-collider detectors would contain highly segmented silicon detectors near the interaction point.

Using a tracking system that included the SVX, CDF measured the B_s mass in the decay $B_s \to J/\psi\phi$. Its first result was superceded by a later analysis that used a larger data sample and improved alignment of the tracking system. The value obtained, $m_{B_s} = 5.3696 \pm 0.0023 \pm 0.0013$ GeV, is in good agreement with the results from the LEP experiments, but much more precise.

The analog of the Λ_c with c replaced by b was expected to have semileptonic decays $\Lambda_b \to \Lambda_c \ell^- \nu X$, where X is some collection of ordinary $u - d$ mesons. The expected non-leptonic decays included $\Lambda_b \to \Lambda_c \pi^-$, $\Lambda_b \to \Lambda J/\psi$, and $\Lambda_b \to pD^0\pi^-$. As early as 1991, the Split Field Magnet Collaboration at CERN's Sp$\overline{\text{p}}$S Collider, observed a significant number of events in the $\Lambda_b \to pD^0\pi^-$ and $\Lambda_b \to \Lambda_c\pi^-\pi^-\pi^+$ channels, though its mass determination had uncertainties of more than 100 MeV. UA1 reported a large branching ratio for $\Lambda_b \to \Lambda J/\psi$, which subsequent data from CDF and LEP experiments contradicted.

The clarity of the semileptonic decay permitted measurement of the Λ_b lifetime even before there was a precise mass determination. The value $\tau(\Lambda_b) = 1.24 \pm 0.08$ ps cited by the Particle Data Group in 1998 is clearly less than the lifetimes of the B mesons.

Exclusive non-leptonic Λ_b decays were finally observed at LEP by DELPHI and ALEPH. DELPHI found four events, three in $\Lambda_c^+\pi^-$, one in $\Lambda_c^+ a_1^-$, with $a_1^- \to \pi^-\pi^-\pi^+$. ALEPH found four events as well, all in $\Lambda_c^+\pi^-$. Unfortunately the mass determinations by DELPHI ($5.668 \pm 0.016 \pm 0.008$ GeV) and ALEPH ($5.614 \pm 0.021 \pm 0.004$ GeV) were only marginally consistent. CDF used exclusively the $\Lambda J/\psi$ mode since it has the superior signature needed for triggering in the high-rate environment of a hadron collider. The signal-to-background ratio was limited: 38 events were seen where background would have given 18. Nonetheless, with high precision tracking the mass was measured to be $5.621 \pm 0.004 \pm 0.003$ GeV.

What is the role of the b quark in weak interactions? Beta decay is described at the quark level by the transition $d \to ue^-\overline{\nu}$. Positron emission is the result of $u \to de^+\nu$. The strangeness-changing semileptonic weak decays (e.g. $\Lambda \to pe\overline{\nu}$)

are described by $s \to ue^-\overline{\nu}$ whose inverse is $u \to se^+\nu$. The Cabibbo hypothesis is that the weak current is really $u \to (\cos\theta_c d + \sin\theta_c s)$. As discussed in Chapter 9, the introduction of a fourth quark makes the Cabibbo angle into a rotation, with the current described by

$$\begin{pmatrix} \overline{u} & \overline{c} \end{pmatrix} \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The V-A structure $\gamma_\mu(1-\gamma_5)$ has been suppressed for clarity. The 2×2 matrix can be viewed either as a rotation of the charge $-1/3$ quarks or of the charge $+2/3$ quarks, though by convention it is usually the charge $-1/3$ quarks that are subjected to rotation.

With the discovery of the b quark it was apparent that the Cabibbo matrix would have to be expanded to a 3×3 matrix. Indeed this possibility had been anticipated by M. Kobayashi and T. Maskawa before even the discovery of the charmed quark. They observed that if there were a third generation, that is a third pair like (u,d) and (c,s), the 3×3 mixing matrix would allow for CP violation.

In order to provide for CP violation, we need a complex term in the interaction $J_\mu^\dagger J^\mu$ where $J_\mu = \overline{U}\gamma_\mu V(1-\gamma_5)D$ is the weak current. If there are n families, U represents the column of n charge 2/3 quarks and D the column of n charge $-1/3$ quarks. The matrix V is unitary and has n^2 complex or $2n^2$ real parameters. Unitarity imposes the conditions $V_{ij}V^*_{kj} = \delta_{ik}$, which give $n(n-1)/2$ complex constraints for $i \neq k$ and n real constraints for $i = k$. Altogether there are n^2 remaining free parameters in V.

It is possible to eliminate some of the complex phases in V by redefining the phases of the $2n$ quark fields. Changing all of the fields by the same phase changes nothing so $2n-1$ phases from V can be eliminated in this way. Thus the number of real parameters characterizing V is $n^2 - 2n + 1 = (n-1)^2$. For two families this gives just one parameter, which is the Cabibbo angle. For three families there are 4 parameters. Now if V were purely real it would be a 3×3 rotation matrix, which is determined by three real parameters. Thus the fourth parameter of V must necessarily introduce a complex component into V, one that cannot be absorbed into a redefinition of the quark fields.

We can represent the Kobayashi–Maskawa (KM) matrix by

$$\begin{bmatrix} \overline{u} & \overline{c} & \overline{t} \end{bmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

In principle, the squares of the various matrix elements can be measured by observing a variety of weak decays. The comparison of nuclear beta decay and muon decay indicates $|V_{ud}| = 0.9735 \pm 0.0008$, while the strangeness-changing decays give $|V_{us}| = 0.2196 \pm 0.0023$. These two are just $\cos\theta_c$ and $\sin\theta_c$ in the Cabibbo scheme. The production of charmed particles in neutrino (or antineutrino) nucleus scattering is proportional to $|V_{cd}|^2$. Data from several experiments lead to a value $|V_{cd}| = 0.224 \pm 0.016$.

The decay of mesons containing b quarks is controlled by V_{ub} and V_{cb}. The best measurements of V_{cb} are obtained by studying the semi-leptonic decays like $B \to D\ell\nu$ and $B \to D^*\ell\nu$. Their decay amplitudes are products of one factor for the leptonic current and one factor for the hadronic current. The leptonic current is as described in Chapter 6. The hadronic current depends on various form factors, for which a theoretical model is required. Heavy quark effective field theory imagines a limit in which the b and c quarks are both very heavy. In a B or D meson then, the heavy quark is just a source of color charge and the light quark doesn't much notice whether it is a b or c quark to which it is bound. This leads to many important simplifications, including relations between otherwise independent form factors. Using HQFT, analyses of data from CLEO II and LEP lead to the result $|V_{cb}| = 0.0402 \pm 0.019$.

The processes arising from $b \to u$ transitions are rarer. For example, while $BR(B^0 \to D^{*-}\ell^+\nu) \approx 4.6\%$, the analogous process $B^0 \to \rho^-\ell^+\nu$ has a branching ratio of about 2.6×10^{-4}. If we estimate crudely, ignoring even phase space considerations, we find $|V_{ub}/V_{cb}| \approx (0.00026/0.046)^{1/2} = 0.075$.

It is possible to measure this ratio in another way, by looking at all semileptonic decays. When the b quark decays to a u, there is much more phase space available for the leptons than when it decays to a c quark, so the highest energy leptons are due to $b \to u$ transitions. By studying the energy spectrum of the leptons, it is possible then to determine what fractions come from the two different quark-level processes. The result is $|V_{ub}/V_{cb}| = 0.090 \pm 0.025$.

As a unitary matrix, V satisfies the condition $VV^\dagger = I$. Wolfenstein showed that it is possible to write a very good approximation for the Kobayashi-Maskawa matrix:

$$V = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

It is assumed that λ is small. This form satisfies unitarity through order λ^3. We see that the data agree with the expectation $|V_{us}| = |V_{cd}|$. The parameter λ has a value near 0.22. From $|V_{cb}| = 0.040$ we conclude that $|A| = 0.83$. Moreover, we know from $|V_{ub}/V_{cb}| = \lambda\sqrt{\rho^2+\eta^2} = |0.090 \pm 0.025|$ that $\sqrt{\rho^2+\eta^2} = 0.40 \pm 0.11$.

A good measurement of $|V_{td}|$ would enable us to determine ρ and η separately. As we shall see, the behavior of the t quark makes it improbable we shall measure $|V_{td}|$ directly. On the other hand, oscillations between B^0 and $\overline{B}^0$ will provide the means to measure $|V_{td}|$ indirectly and to test the CKM matrix thoroughly, as discussed in Chapter 14.

When a photon interacts with a quark, the quark type does not change. However, photon emission can occur during a weak interaction. This allows the transition $b \to s\gamma$. The b quark can emit a virtual W boson and become a u, c, or t quark, which reabsorbs the W to become an s. During this transition, the photon is emitted by one of the particles. For obscure reasons, this process is usually referred to as the penguin diagram. In 1993, the CLEO Collaboration established the decays $B^0 \to K^{*0}\gamma$ and $B^- \to K^{*-}\gamma$, which are evidence of this process **(Ref. 11.10)** . The decay $B \to K\gamma$ is forbidden by conservation of angular momentum.

The decay $B \to K^*\gamma$ was discovered among 1.4 million events taken with the CLEO-II detector at the $\Upsilon(4S)$ resonance. Because even at its peak the $\Upsilon(4S)$ accounts for just 20% of the cross section, studies were made off resonance to understand the backgrounds. For the decay $B^0 \to K^{*0}\gamma$, the K^{*0} was detected as $K^\pm\pi^\mp$. The photon was observed in the electromagnetic calorimeter comprising 7800 crystals of CsI. For the charged B decay, the K^* was seen as $K_S^0\pi^\pm$ or $K^\pm\pi^0$. Altogether 13 signal events were found with an estimated background of two events. The neutral and charged decays were found to have similar branching ratios, near 5×10^{-5}.

The process $b \to s\gamma$ is of interest because it tests our understanding of weak interactions and because the analogous processes $b \to s\ g$ and $s \to d\ g$, where g is a gluon, play important roles in non-leptonic weak decays. The latter can contribute to the decay $K \to \pi\pi$ when the gluon turns into $d\overline{d}$ or $u\overline{u}$. Thus $s\overline{d} \to d\ g\overline{d} \to s\overline{u}u\overline{d}$. Because the gluon has $I = 0$, $s \to dg$ is a $\Delta I = 1/2$ process. This helps to explain the dominance of $\Delta I = 1/2$ in non-leptonic strangeness-changing decays. Moreover, penguins contribute to the CP violation parameter ϵ'/ϵ in $K \to \pi\pi$.

EXERCISES

11.3 Suppose the quark–antiquark potential obeys the power law $V(r) = ar^\nu$. Show that the binding energies vary with the quark mass as $E \propto m^{-\nu/(\nu+2)}$ and that the density at the origin $|\phi(0)|^2$ for an s-wave state varies as $m^{3/(\nu+2)}$. Given that the splitting $\Upsilon' - \Upsilon$ is nearly identical to that for $\psi' - \psi$, predict $\Gamma(\Upsilon \to e^+e^-)$ from $\Gamma(\psi \to e^+e^-)$ if the charge of the new quark is $-1/3$ or is $+2/3$.

11.4 Show that $|\phi(0)|^2$, the s-wave wave function at the origin squared, is related to the average force by

$$|\phi(0)|^2 = \frac{m}{2\pi} < F >$$

Hint: write the Schrödinger wave equation for the radial wave function, u and multiply by u'. Integrate the result from $r = 0$ to $r = \infty$.

11.5 Use the results of Exercise 6.5 to determine the e^- spectrum in the decays $b \to ce^-\overline{\nu}$, $b \to ue^-\overline{\nu}$. Take $m_b = 5$ GeV, m_c= 1.5 GeV, m_u= 0.3 GeV.

11.6 * Suppose that the $b\overline{b}$ or $c\overline{c}$ interaction can be represented approximately by a nonrelativistic Schrödinger equation:

$$\left[-\frac{1}{2m_{red}}\nabla^2 + V(r)\right]\psi = E\psi$$

where $m_{red} = m/2$ is the reduced mass. Then the energy levels are spin-independent so ${}^3S_1$ and ${}^1S_0$ are degenerate, as are ${}^3P_{2,1,0}$ and ${}^1P_1$, etc. Now consider as perturbations the spin dependent forces

$$\begin{aligned}
&\mathbf{L}\cdot\mathbf{S}\, V_{so}(r)\\
&\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\, V_{spin-spin}(r)\\
&\frac{3\,\boldsymbol{\sigma_1}\cdot\mathbf{r}\,\boldsymbol{\sigma_2}\cdot\mathbf{r}-\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}r^2}{r^2}V_{tensor}(r) \equiv S_{12}V_{tensor}(r)
\end{aligned}$$

Here $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$ is the total spin, and $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the quark and antiquark spin operators.

a. Which degenerate states are split by each of the interactions? Which nondegenerate state are mixed by the interactions?

b. Use the relations (try to prove them, too)

$$\begin{aligned}
&< J, L = J+1, \quad S=1, M|S_{12}|J, L=J+1, \quad S=1, M >= -2\frac{L+1}{2L-1}\\
&< J, L = J, \quad S=1, M|S_{12}|J, L=J, \quad S=1, M >= +2\\
&< J, L = J-1, \quad S=1, M|S_{12}|J, L=J-1, \quad S=1, M >= -2\frac{L}{2L+3}
\end{aligned}$$

to analyze the observed splittings of the ${}^3P$ states in the Υ and ψ systems. Here $|J, L, S, M>$ is a state with total angular momentum J, orbital angular momentum L, total spin angular momentum S, and $J_Z = M$. [See J. D. Jackson, "Lectures on the New Particles" in *Proc. of Summer Institute on Particle Physics, Stanford, CA, Aug. 2-13, 1976*, M. Zipf, ed.]

11.7 * The relation between the standard relativistic Lorentz invariant amplitude, $\mathcal{M}$, (the usual Feynman rules generate $-i\mathcal{M}$) and the conventional scattering amplitude of potential theory is

$$f = -\frac{1}{8\pi\sqrt{s}}\mathcal{M}$$

where s is the square of the center of mass energy. The center-of-mass differential cross section is $d\sigma/d\Omega = |f|^2$. In potential theory, the Born value for f is

$$f = -\frac{m}{2\pi}\int d^3r e^{-i\mathbf{p}'\cdot\mathbf{r}}V(r)e^{i\mathbf{p}\cdot\mathbf{r}}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta. Two body scattering can be treated analogously with the modification $m \to m_{reduced}$. If the particles have spin 1/2, we generalize the wave function to $\psi(r)\chi_1\chi_2$, where χ_1 and χ_2 are two-component spinors. Thus

$$f = -\frac{m_{red}}{2\pi}\int d^3r \chi_2'^\dagger \chi_1'^\dagger e^{-i\mathbf{p}'\cdot\mathbf{r}} V(r) e^{i\mathbf{p}\cdot\mathbf{r}}\chi_1\chi_2$$

Suppose $\mathcal{M}$ has the form of vector exchange but with some more general dependence on momentum transfer:

$$\mathcal{M} = \overline{u}(p_4)\gamma_\mu u(p_2)\ \overline{u}(p_3)\gamma^\mu u(p_1)\ \tilde{V}(p_1 - p_3)$$

Show that the spin dependent potential is, to leading order

$$V(r) = V_0(r) + \frac{3}{2m^2}\frac{1}{r}\frac{dV_0}{dr}\mathbf{L}\cdot\mathbf{S} + \frac{1}{12m^2}S_{12}\left(\frac{1}{r}\frac{dV_0}{dr} - \frac{d^2V_0}{dr^2}\right) + \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{6m^2}\nabla^2 V_0$$

where $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$ is the total spin and

$$\int d^3r\, V_0(r) e^{i\mathbf{q}\cdot\mathbf{r}} = \tilde{V}(q)$$

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